Prospects for the Detection of Planetary Rings around Extrasolar Planets

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Contents

1 Introduction

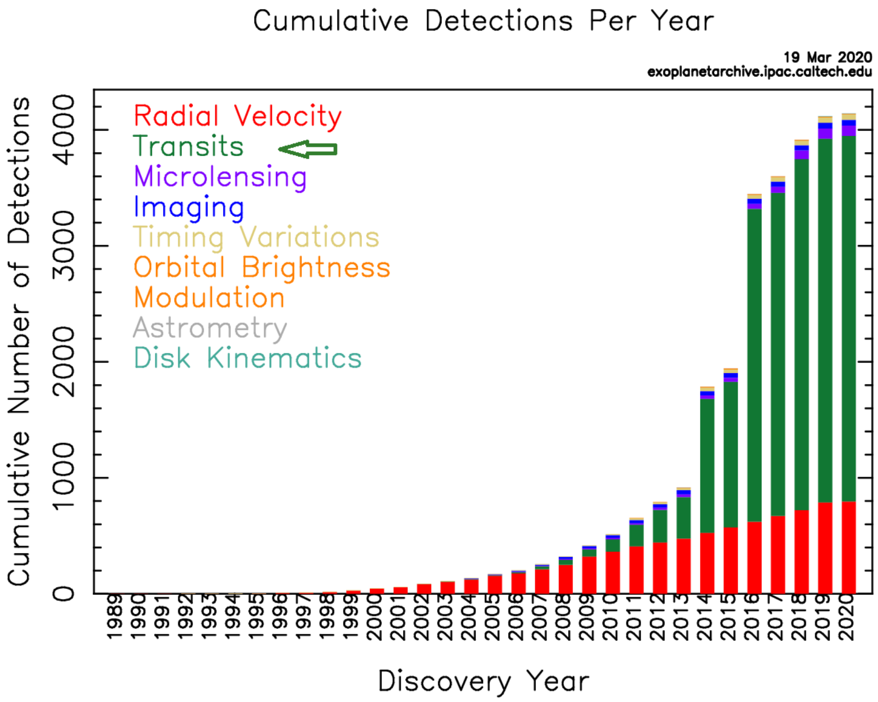
Back in the 1600's Galileo Galilei looked to the sky with his telescope and observed Saturn. The telescope was quite inaccurate and Galilei thought that he saw Saturn with two big moons on both sides of it. Christian Huygens realised that it was actually a ring around Saturn. For couple centuries the general belief was that Saturn's rings were solid but in 1859 James Clerk Maxwell proved that solid rings would not be stable but break apart. He proved that the rings must consist of small particles which all orbit Saturn. Finally in 1895 James Keeler proved that Maxwell was right with his assumptions by showing that different parts of Saturn's rings reflect light with various Doppler shifts [60]. [32]

Extrasolar planets, also shorter called exoplanets, are one of the newest and most rapidly growing fields of astronomy today. Exoplanets are planets which orbit a star outside of our Solar system, in other words they orbit other stars than the Sun. [1] The first ever detection of an exoplanet was back in the year 1992 around a pulsar [61] but the first exoplanet which orbited a normal star was discovered in the year 1995 [20] and as of today 4183 exoplanets have been detected. [2] The majority of extrasolar planets are found with the transit method. In figure 1 (a) the cumulative number of all exoplanet detections is shown color coded for the method of detection and in (b) the range of semi-major axes of exoplanets and how it has increased over years.

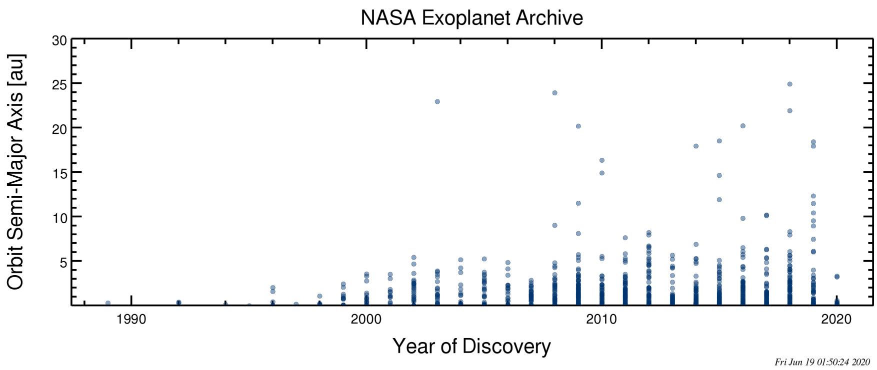
There are different techniques how exoplanets can be detected. Discoveries were first made mainly with the radial velocity technique until the Kepler Telescope became active in the year 2009 and multiplied discoveries with the transit method. [7] As we see the detected distances between planets and stars have been also increased with improved instruments. In the future there will be many new campaigns to discover new exoplanets when the James Webb Space Telescope (JWST) [45], PLATO and ARIEL [44] will be launched. By new campaigns it is expected that exoplanet discoveries will be multiplied.

Planetary rings in the Solar system consist of many small bodies orbiting their planets in size from micrometers to tens of meters. Their composition varies from water-ice to silicates. In our own Solar system each giant planet has its own ring system and therefore it is reasonable to expect that planets orbiting other stars could have rings too. [3]

Astronomers have been searching for rings around exoplanets for years. [62][63] Some researches have been made but no ring systems have been confirmed around exoplanets yet. Barnes et al. (2004) [3] investigated if exo-rings could be detected from transit light curves and how they would affect on light curves in different orientations. This research has been used by other exo-ring researchers since then as basis for further study. Schlichting et al. (2011) [10] examined the nature of rings that could exist around exoplanets. In general they investigated parameters



(a)



(b)

Figure 1: Cumulative detections per year (a) and Annual detections versus exoplanet semi-major axes (b). Figures from NASA [6]

which would affect the chances for detecting of rings and why rings around exoplanets should exist in first place. Zuluaga et al. (2015) [4] proposed a method to find candidates for exoplanets having rings which based only on measurement of basic transit parameters and therefore avoids time-taking fits of ring models. They also studied in detail the geometry of a ring transit. All these researches have been used as base for this thesis.

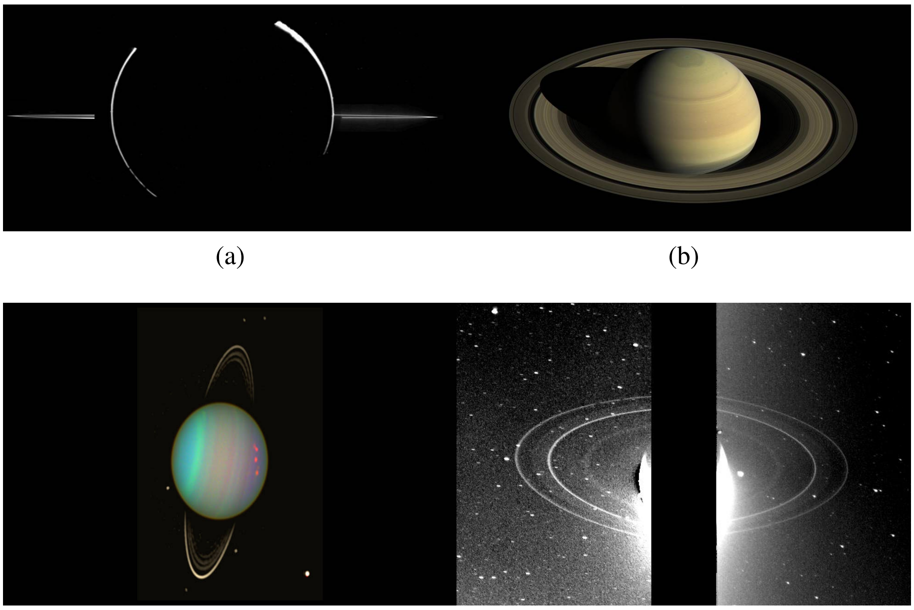
The structure of this thesis is as follows; At first we present the background of exoplanet and exo-ring research, light curves and theoretical aspects of ring systems. Then we get acquainted with the generation of synthetic light curves. In section four we discuss about the light curve data available online and how the exoplanet candidates for this thesis have been selected. Finally we present light curves of selected exo-ring candidates and examine and compare them. The ultimate objective of this thesis is to search literature of the topic and screen it to the study, examine the databases, review it and also discuss about it and present the models made for the thesis. In practical point of view first we search for the light curve data available online, examine it to find features of rings and compare the findings to the modelled synthetic light curves and discuss results.

2 Background

2.1 Rings in the Solar System

As told in the previous chapter every giant planet of Solar system owns a ring system. They all vary from each other in composition, thickness and size. The most studied ring system is definitely Saturn's ring system because of the Cassini orbiter mission.[51] It contains mostly water-ice but also rock while the main rings are nearly just water-ice. The sizes of particles vary from micrometers to even ten meters. The ring system is really thin starting from only one meter to kilometer. With current knowledge Saturn's rings could not have been existing more than 100 million years and may vanish in the next 100 million years. In a long term evolution Saturn's magnetic field is influencing the rings where collisions between particles happen and eventually those particles "rain" to the surface of the planet. [46] Saturn's rings' origin is most likely from colliding icy satellites around Saturn. [5]

The rings of Saturn were found in 1610, while Uranus' were discovered 1977 by James L. Elliot, Edward W. Dunham, and Jessica Mink [23], Jupiter's by Voyager 1 in 1979 [22] and Neptune's by Patrice Bouchet in 1984 [24]. Compared to other giant planets Jupiter, Uranus and Neptune, Saturn's rings are completely different. They can even be seen with good binoculars from the Earth, unlike other giant planets' rings. The reason for that and why Saturn's rings were found easier than other gas giants' is the optical thickness. Optical thickness describes the ability of medium to absorb and reflect light. When medium has higher optical thickness an average photon can not pass it without being absorbed. That kind of medium is also called opaque or optically thick. [41] While other giant planets' ring particles are orbiting with big distances between each other, Saturn's ring particles are packed a lot denser. Also Uranus has dense rings, but they are narrow unlike Saturn's and therefore can not be observed without better instruments. [64] Therefore it would be reasonable to think that if we want to detect rings around extrasolar planets in other star systems with current optics and techniques they must be optically thick as Saturn's main rings. Otherwise rings would not affect the transit light curves noticeably.



(a) (b)

(c) (d)

Figure 2: Rings of Jovian planets Jupiter (a), Saturn (b), Uranus (c) and Neptune (d). Source of figures: NASA

There is also a possibility that having a stable and pretentious ring system is quite rare. For example if the typical age of a ring system is around 100-200 million years maximum we would need to be lucky to detect one right at the observation time since the age of a planet can be billions of years. On the other hand it might be possible that even if a planet lost its ring system it could gain a new one for example in the destruction of a satellite. [26]

2.2 Conditions for Having Rings

Not every planet can have a stable ring system. If a planet is close to their host star there will be effects of Poynting-Robertson drag and viscous drag from the planet's exosphere which affect ring stability negatively. [48] Also the local equilibrium temperature affects what can be the composition of the rings. If a planet orbits its host star close enough, for example ice composed rings would evaporate because of the heat. [10] According to Gaudi et al. (2003) [48] semi-major axis of the planet can not be smaller than

$$a \approx \left(\frac{L_\star}{16\pi\sigma T_{sub}^4}\right)^{1/2} = 2.7\left(\frac{L_\star}{L_\odot}\right)^{1/2} AU \tag{1}$$

for ices to exist. That distance is later referred as "ice-border". $L_\star$ is luminosity of the host star and $L_\odot$ the luminosity of the Sun. Luminosity of the star can be derived from equation 2 [1]

$$L = 4\pi R_\star^2 \sigma T^4, \tag{2}$$

where $R_\star$ is the radius of the star, σ is Stefan-Boltzmann constant and T is the effective temperature of the star. Therefore planets with orbital radius smaller than that can not have ice composed rings but could still have rock composed rings.

The circumstances for a planet to have rings vary a lot. Probably the largest variable is the size of the host star. If the host star is large and hot its planets should orbit it at a bigger distance than in the case of a cold and small star. Otherwise the planet can not have stable rings around it. The challenge here with current data and optics available is that the majority of the exoplanets found to this day do orbit their host star with semi-major axis smaller than 1 AU as seen in the figure 3.

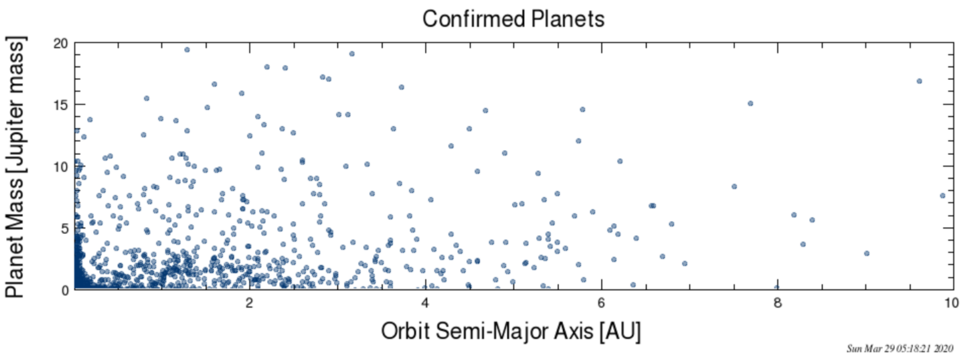


Figure 3: Confirmed planets' semi-major axes and masses. Figure from NASA [6]

Therefore the probability of those exoplanets to have rings is very small (from our current knowledge). That means that if we want to find the best exoplanet candidates which could have rings we need to keep an eye on the planet's semi-major axis and host star size relation. Rationally thinking the best possible and ideal circumstances would be that the planet orbited its host star with a radius of few AU and the size of its host star was near the size of the Sun as all gas giants have in our Solar system. But as seen from figure 3 there are not that many extrasolar planets found orbiting its host star with a distance of several AU's. Therefore in this thesis we need to also consider if planets even with orbit of less than 0.5 AU had rings.

2.3 Probability of Detecting Rings

Nowadays almost every new exoplanet discovered is found with the transit method when earlier years radial velocity was the main discovery method as seen in previous figure 1 (a). The transit method gives us a possibility to examine stars' light curves where from it is possible to try to find features of ring systems around transiting planets.

That we could see signs of rings on transit light curves the data should be high enough in quality. If the signal-to-noise ratio is higher the quality of the data is also higher and then we would see features of rings easier in the light curves. For example if the signal-to-noise ratio S/N is 20 then the noise $N = S/20 = 5\%$ of the signal. The exact S/N-ratios can be derived from light curves but the estimation for the quality of data can roughly be seen with naked eye from the light curves and for the purpose of this thesis we do not need to derive the exact values. Also the amount of data points do affect the readability of the data. For the ring transit a big amount of data points for short time is needed because otherwise ring features could not be seen precisely.

3 Detection of Extrasolar Ring Systems

Extrasolar planets can be detected with many different techniques where from transiting is by far the most used method. The basic idea of transit method is that an object, in this case exoplanet travels between the observer and the star. When that happens the luminosity of the star drops for a moment and if it happens repetitively and regularly we can assume that a planet orbits that star. [1]

The same rule applies for ringed exoplanets also because the orientation of rings should be the same on every transit and therefore the transit pattern should be the same every time. In other words features of rings should be seen when exoplanet enters the transiting zone and exits from it and that should happen on every transit that we could assume that rings exist on that particular planet. [4][27]

The Zuluaga method [4], which will be used to search for exo-ring candidates, is based on basic transit parameters as depth in transit light curve, duration of transit and the shape of the transit pattern and therefore because of its simplicity it suits well for searching exoplanets with rings from large databases. The method has three strategies to filter exoplanets to find the best candidates among all. Those strategies are:

1. Look for confirmed exoplanets detected by transit method which have abnormally low densities. To be able to obtain the density, exoplanet have to be detected also with other method, for example radial velocity method.

2. Look for transiting objects that have been marked as false positives because of their abnormally large transit depths.

3. Look for transit signals for which a negative asterodensity profiling effect is observed. Transiting exoplanets allow one to determine the mean stellar density of the system's host star with few small idealized assumptions. Asterodensity profiling does compare that density to a density which have been obtained independently and checks the validity of the assumptions. [49][4]

In this thesis rings on exoplanets are searched by using strategy number one (1). The reason to search for rings from abnormally low density planets is that because of rings a planet's radius could be false measured as too big. Then after further independent researches when planet's mass is measured, the density of the planet gets false measured because of overly big planet radius.

3.1 Transit Light Curves of a Ringed Planet

On the following figure 4 we can see a schematic drawing how a transiting planet is expected to show up in a light curve. On this example b is transit depth, $a1$ is planet's transiting time on the edge of the star and $a2$ is the time planet transits across the star. Flux presented in all light curves in this thesis are so called "normalized fluxes". It means that star's normal flux without any interference is set to be 1 (100%). When planet transits between us and a star, the flux drops. If star's flux is for example 0.97 when transit happens, it means that the flux is 97 % of star's normal flux.

Duration of transit and transit depth do vary for different stars and there are couple parameters which affect the duration and depth of a transit. At first the size of a planet affects the transit depth. For a larger planet the transit depth becomes deeper. On the other hand when the observed star's size is larger the transiting time also grows up but the change of brightness of a star is smaller. So from that we can figure out that if the observed star is huge and a planet which is small in

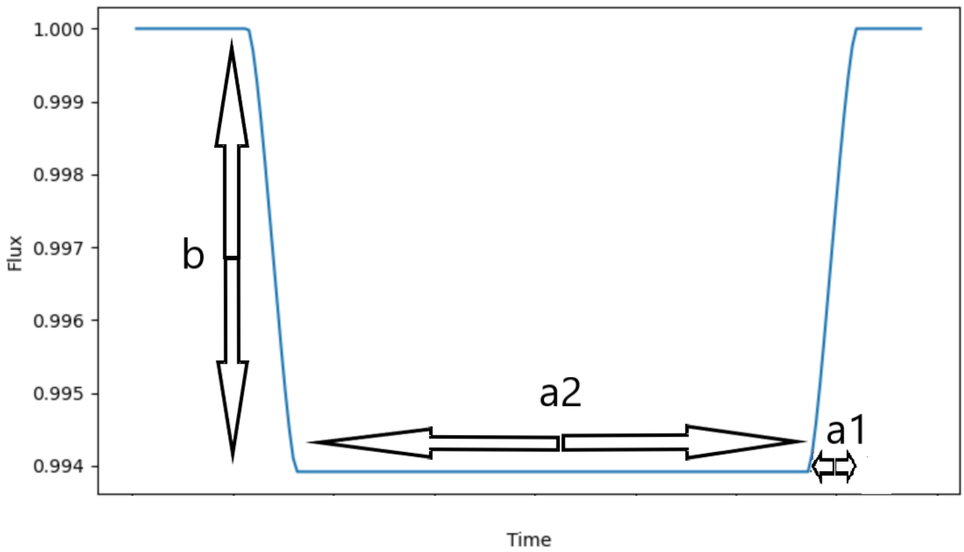


Figure 4: Synthetic transit light curve of a planet without rings. Time steps in this and all upcoming synthetic light curves are in same scale. The definition of one time step is explained in section "Implementation of synthetic light curve"

size orbits it, we will have big difficulties to discover transits from the light curve. [11] A figure to demonstrate the effect of size of the planet will be presented later in section 4.3.

In real light curves obtained from observations the main phase of a transit is not that flat like in figures 4 and 5. Reason for that is limb-darkening. Limb-darkening is an optical effect which can be seen in the stars. Reason for limb-darkening is that the light we see on the edges is from the layers close to the surface of the star. Those areas are cooler and radiate less than layers from deeper parts of the star. In practice limb-darkening causes the central parts of the stellar disk to appear brighter than edges. Therefore the effect of a transit as transit depth is greater in central parts than on the edges.

The light curve of a ringed planet would look some way different than the light curve of a planet without rings. There are several reasons for that: At first the transit depth of a ringed planet would be deeper than for the same planet without rings. That is because more area is preventing light from a star to reach the observer's eye and therefore dimming of the star is bigger. Secondly the start and the end times of the transit may be different. When a planet enters and leaves the transit zone, the most likely the first and the last part which transits are the rings. In the general case rings are not as optically thick as the planet itself so the effect which it causes on brightness of a star in that case is not that big that it is when the planet itself transits. [4] In other words practically the rings do not block the same percentage of star's light compared to a planet itself. Therefore if we want to find evidence for ringed exoplanets we should try to find the features characteristic for rings in the light curves.

On the figure 5 we can see a schematic drawing how ringed planet should appear in light curve. There ar is the time when rings of planet enter and leave transiting zone. This area of transit pattern is later called a "transit wing". The time when the planet itself enters and leaves the transiting zone is $a1$ and $a2$ is the time when the planet and its rings are both transiting across the star. As previously b is the transit depth. There is also a possibility that rings are orientated towards us in the way that rings are not transiting before the planet does. In that case we can not see rings to cause transit wings for the transit pattern but they still affect the depth of the transit pattern.

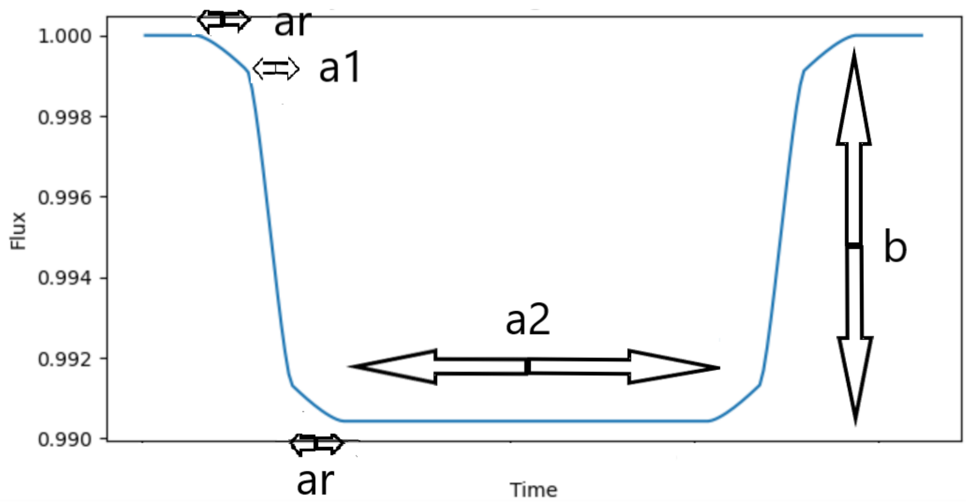


Figure 5: Synthetic transit light curve of planet with rings

3.2 Ring Orientation

We expect to observe rings in different angles and orientations depending from our observing position towards the ring system. There are two ways how rings can be tilted; The first way is rings' angle. Rings' angle θ can vary between 0 and 360 degrees as seen in figure 6. If we change rings' angle it does not affect the absorption of the rings because the apparent area of the rings stays constant all the time. However it affects the shape of the transit pattern from the edges of a transit while the transit depth stays constant. [16] For example orientation (a) would give us the transit pattern presented earlier in figure 5 in shape, but orientation (c) would shorten the transit wings remarkably while the transit depths for both orientations would be exactly the same.

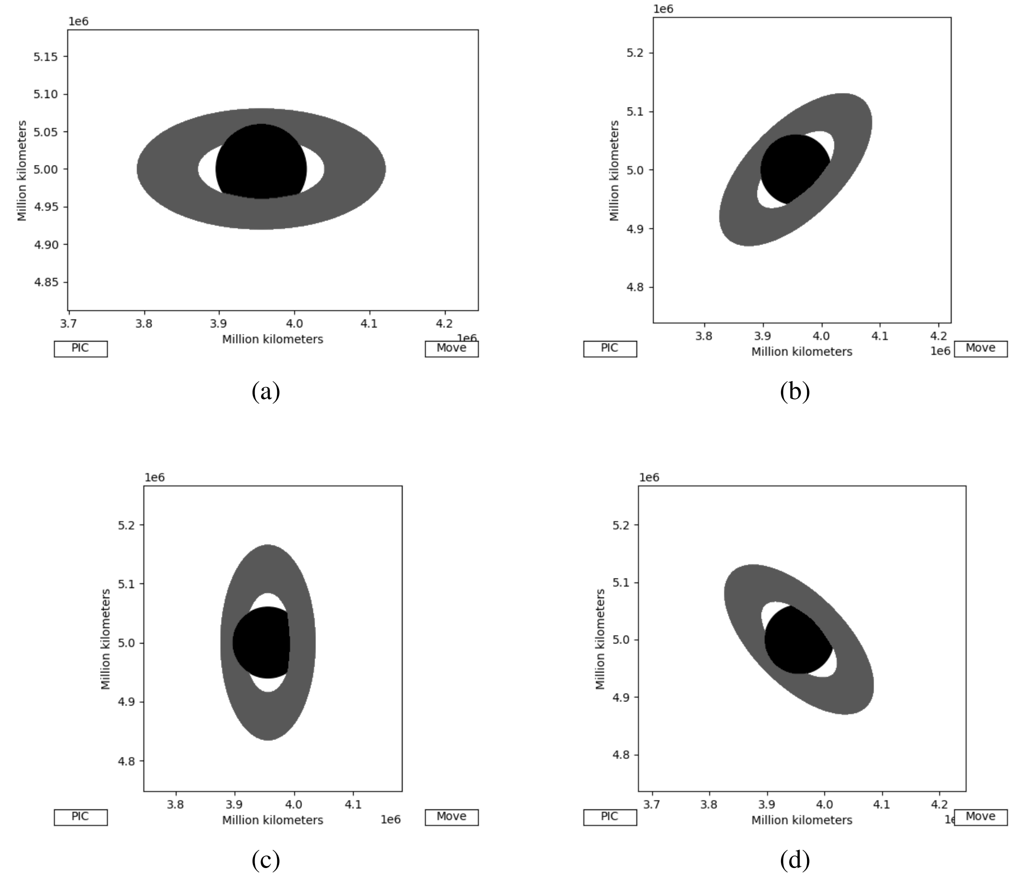


Figure 6: Schematic example to show ring angle θ at $0°, 45°, 90°$ and $135°$, respectively. Ring system shown in 2D-plane where X- and Y-axes are expressed as millions of kilometers

The second way how rings can be tilted and the more important factor is the inclination of the rings i_R towards us if we want to consider the absorption of the rings. The inclination angle can vary also between 0 and 360 ° but because of geometrical symmetry we restrict the angle to vary between 0 and 90 °. In figure 7 we can see few possible inclination angles. In other words if rings of observed planet are face-on $i_R = 0°$ it makes transit depth deeper than planet with $i_R = 45°$. [16] However the possibility that we would observe the planet with ultimate $i_R = 0°$ or $i_R = 90°$ is tiny.

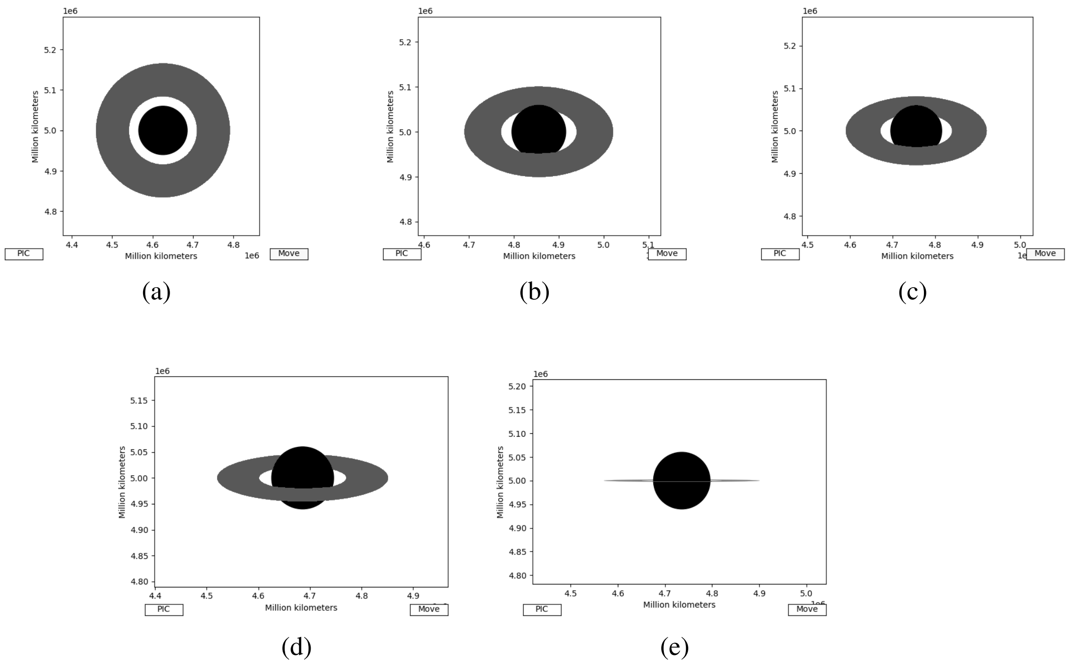
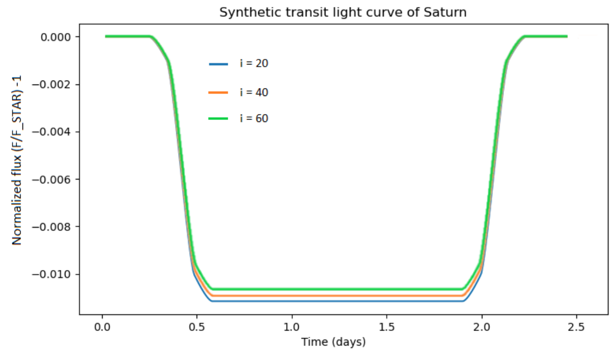
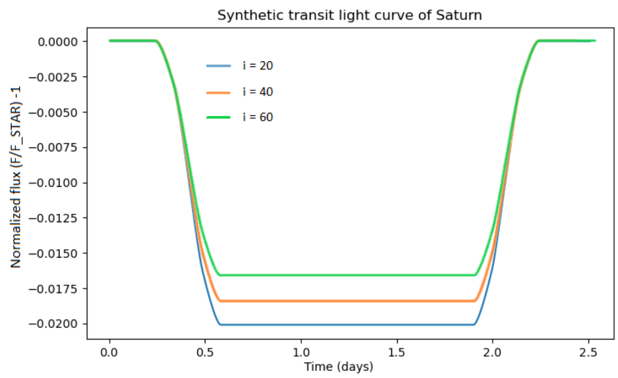


Figure 7: Schematic example to show ring inclination i_R at $0°, 45°, 55°$, $70°$ and $90°$, respectively.

The smaller the inclination angle is, more the rings affect on transit depth of the transit light curve. The change of transit depth is greater if optical depth τ is higher. For low optical depth the change in inclination angle does not change the transit depth remarkably. In figure 8 (a) we can see Saturn size planet with rings transiting the Sun with three different inclination angles when $\tau = 0.1$. In figure 8 (b) we can see the same transit with same inclination angles when $\tau = 0.5$.



(a)



(b)

Figure 8: Blue line = $i_R = 20°$, orange line = $i_R = 40°$, green line = $i_R = 60°$. Optical depth τ = 0.1 in (a) and 0.5 in (b).

Rings' effective absorption factor β is dependent on the inclination angle of the rings i_R and the optical depth τ with following equation [3][4]:

$$\beta = 1 - e^{\frac{-\tau}{cos(i_R)}}. \tag{3}$$

Practically it gives the fraction of the stellar light rings do absorb. τ in equation 3 is defined as the vertical optical depth. Amount of light to surpass the rings decreases exponentially along the oblique path what for the optical depth is given as $\tau/cos(i_R)$. For example if $\beta = 0.20$, then 20 per cent of the light is absorbed by the rings and 80 per cent come through them. If τ or i_R increases it will also increase the value of β.

3.3 Producing a Synthetic Transit Light Curve from a Model

From now on Synthetic Light Curve will be shortened as SLC. To make the estimations and assumptions of ring systems more valuable and reliable this thesis derives SLC's made with a Python-program [Appendix, figures 33-40]. The fundamental idea of the program is to model the transit light curve of a target if there was not any interference in the optics, atmosphere and interplanetary space and then produce a theoretical light curve with any wanted parameters.

The parameters which can be changed are following:

1. Radius of the host star R_{star}.
2. Radius of the transiting planet R_{planet}.
3. Radius of the inner and outer rings R_{irings} and R_{orings}.
4. Optical depth of the rings τ.
5. Inclination of the rings i_R.
6. Orbital period of the planet (days).
7. Orbital mean distance of the planet (AU's).

There has also been made some assumptions and approximations to make the modelling simpler:

1. Rings are homogeneous and uniform.
2. Planet system transits horizontally along the x-axis i.e. planet system orbits along the orbital plane of the planet. We set the orbital plane in the way that the planet system transits from center parts of the star. Geometrically it would be rare that a planet system would transit the star only touching its edges. The case like that can be ignored by its rareness. Therefore rings do have two intersection points with the star in 2D-plane (unless the ring system is gigantic compared to the star) which simplifies the modelling.
3. Planet and star rotation have been ignored as well star spots and other interferences in stellar disk.
4. Limb-Darkening has been neglected.

As mentioned earlier the flux presented in all SLCs is so called normalized flux. Normalized flux is produced by dividing star's flux during transit steps with its normal flux when transit is not happening. Star's flux during the transit can be determined geometrically with intersection areas of a star, planet and rings by subtracting the intersection areas from the star's total surface area. Intersection areas in SLC's have been defined with integrals; First integral is between a planet and a star (circle-circle integral, Appendix A) and the second integral is between the rings and the star and between the rings and the planet (ellipse-circle integral, Appendix B).

One time step in all SLCs is defined the following way; When running the program, the planet system has a specified starting point before the transit happens. On one time step the planet system moves horizontally with a specified amount of kilometers (for all SLCs chosen to be $1/10$ of the planet's radius). By shortening the distance of the step the light curve becomes more accurate but takes more time to produce and vice versa. If a one time step is not short enough, the features of the rings can not be detected from the light curve. The chosen amount of km is noticed to produce high enough quality light curves and therefore that is used for light curves. These time steps have been then converted to days by using planets' known orbital periods and distances from their host stars. To do that at first we need to calculate the orbital speed of the planet which we basically get from

$$V_o = \frac{Orbitallength}{Orbitalperiod} = \frac{2\pi d}{t}, \tag{4}$$

where d is the mean distance of the planet and the star in kilometers and t is time of the orbital period in seconds. In reality the orbital speed can vary and the difference is bigger if orbital distance varies i.e. eccentricity of the orbit is bigger. Now we assume that orbital speed does not change in time. To convert orbital speed to the time steps we need to divide it from the distance s the planet system travels in one step.

$$Timestep = \frac{s}{V_o}. \tag{5}$$

The result is given in seconds and can be converted to days by multiplying with 86400. SLC is produced in a following way step by step:

1. Run the program and enter all parameters.

2. Visual screen opens where one can see a star and a ringed planet. One can move a planet system in its orbital plane by pressing the "Move" button and produce a light curve by pressing the "Pic" button. Planet has been approximated to orbit its host star on a straight line during the transit since it simplifies the modelling and error it causes is negligible.

3. For every press on the "Move" button program calculates the intersection area between rings and the star and also between the planet and the star. Intersection area between rings and the planet stays constant for the whole transit. Intersection areas are integrated with integrals presented in Appendix A and B. On every step if the planet and the star have a shared area, the planet blocks the star's light entirely from the intersection area. Rings are not blocking light from the star entirely and it needs to be taken into account with rings' effective absorption factor β which was introduced earlier. The ring area which has intersection with the star is multiplied with β factor which gives rings' real effectiveness for a transit. Then after every step that the planet system travels the effective intersection areas are saved to the lists which can be dealt and plotted.

4. After pressing the "Move" button for that many times that a whole transit has occurred one can produce a light curve for the whole transit with the "Pic" button. The program plots a normalized flux in respect of time in days.

3.4 Implementation of SLC

As stated before the basic idea of producing the SLC was to model the transit light curve as there were not any disturbing elements and then compare those "perfect" light curves to the real ones obtained from observations. The program which creates SLC has both visual and mathematical aspects. Visually it shows the star, planet system and the whole transit step by step. At the same time when transit is happening in the visuals the program also defines the intersection areas of a planet system and a star and produces the light curve.

We discussed the possible ring angle θ earlier. For the SLCs we assume that $\theta = 0°$. Reason for that is found when we examine known moons and planetary rings. Generally satellites of a planet as moons and ring material orbit near of their planet's orbital plane. The reason for that is found from the birth of the Solar system when the Sun formed and all additional materials was left in the disk revolving around the Sun. In time that material condensed to form planets and moons, all from the same disk and therefore in the same orbital plane. In time mainly because of collisions between each other the orbital paths of planets and moons may have changed some but in general they are still nearly in the same plane. It means that when we observe other star-planet systems, the planets' supposed rings generally are formed in the orbital plane and therefore θ is close to $0°$. In our Solar System Uranus is an exception: It's rings are tilted about 90 °. Reason for that is not known for sure but the most probably Uranus had a collision with other big object in the past which caused its tilting. [54]

Saturn has an ideal ring size to be searched for from exoplanets because wider rings can leave more distinct signs to light curves than narrow ones. According to Colwell et al. [43] and NASA [17] Saturn's rings have a structure shown in table 1.

Table 1: **Saturn's ring system**

Equatorial radius = 60 268 km		
Ring name	Location from Saturn's center (km)	Optical depth
D-ring	66,900-74,510	10^{-5}
C-inner edge	74,658	0.05-0.35
C-outer edge	91,975	0.05-0.35
B-inner edge	91,975	0.4-2.5
B-outer edge	117,507	0.4-2.5
Cassini division	117,507-122,340	0-0.1
A-ring	122,340-136,780	0.4-1.0
F-ring	139,826	0.1
G-ring	166,000-173,000	10^{-6}
E-ring	180,000-480,000	10^{-6}

As seen in the table 1, the optical depth varies a lot in different regions of the rings. If we averaged the optical depths of Saturn by area between D-ring's inner edge to the A-ring's outer edge it would give us $\tau = 0.26 - 1.28$ where from average is ≈ 0.77. We notice that the optical depth of the rings is significantly lower for G-ring and E-ring. With that low optical depths those regions would not affect the light curve at all and therefore they are ignored. These assumptions mean that rings' outer edge in the model is approximated to be 140 000 km from the center of the planet and the inner edge is approximated to be in a distance of 7000 km from the surface. Then as known Saturn's obliquity (the angle between its rotational axis and its orbital axis) is about 27 degrees. [52] It means that Saturn rings' inclination angle is about 63 degrees if we observed it in its orbital plane. However Saturn has seasons just like the Earth has and during the seasons the inclination angle of Saturn rings changes when observing from the Earth. [69] However we study Saturn's rings with their inclination angle of 63 degrees, because it shows the features of rings better than bigger angles. When applying these parameters we get SLC seen in figure 9.

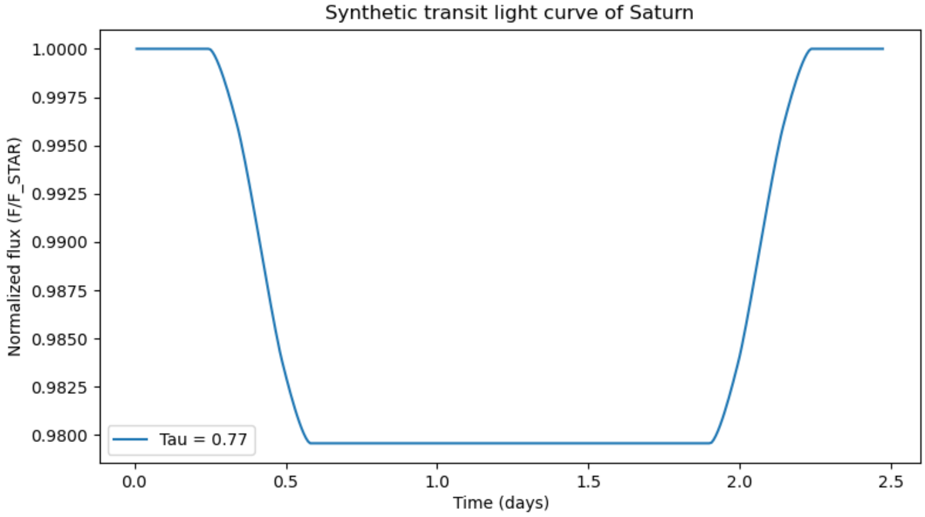


Figure 9: SLC of Saturn with $i_R = 63°$, $\tau = 0.77$, $R_{irings} = 7000km$ and $R_{orings} = 140000$km.

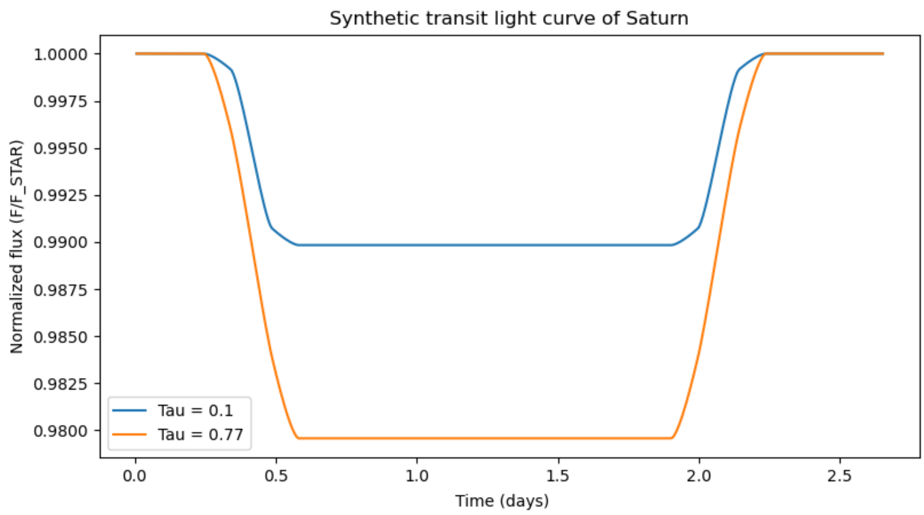


Figure 10: SLCs of Saturn with real parameters ($\tau = 0.77$, $i_R = 63°$) and changed parameters ($\tau = 0.1$, $i_R = 50°$).

We can notice that the ring features in the figure 9 are not clearly seen. It would be unlikely, if not impossible, for an external observer to detect rings of

Saturn in transit. For the purpose to get the ring features out more clearly we examine Saturn's rings if they had $\tau = 0.1$ and $i_R = 50°$. The result can be seen in figure 10.

As we can clearly see the features of rings are much easier to see with changed parameters. We can notice that with larger τ and i_R the transit depth is a lot deeper but transit wings on the edges of a transit can be hardly seen. After all, transit wings are the most obvious way to detect rings from a transit and therefore we will use lower values for τ and i_R to show features of rings more clearly. Also we can assume that the rings of Saturn have mean τ fairly over an average if we compare it to the mean optical depth of other ringed Jovian planets. For example mean optical depth of Neptune's ring system is $\sim 10^{-4}$ [65]. Hence for examining exoplanets' light curves we set $\tau = 0.1$ and $i_R = 50°$.

Finally we need to take into account the effect of the expected ring system for the light curve. If for example a Saturn size planet had ring system with values $\tau = 0.1, i_R = 50, R_{orings} = 140000km$ with basic geometry and using absorption factor β rings in that kind of system would increase the blocking area with $\approx 35\%$. Therefore to get more accurate results we will use planetary area of 65% of the real measured area.

On figure 11 we can see an example light curve of a Saturn size planet transiting Sun step by step which brings out different phases of a transit and the ring features better and also gives a possibility to see how SLC is produced.

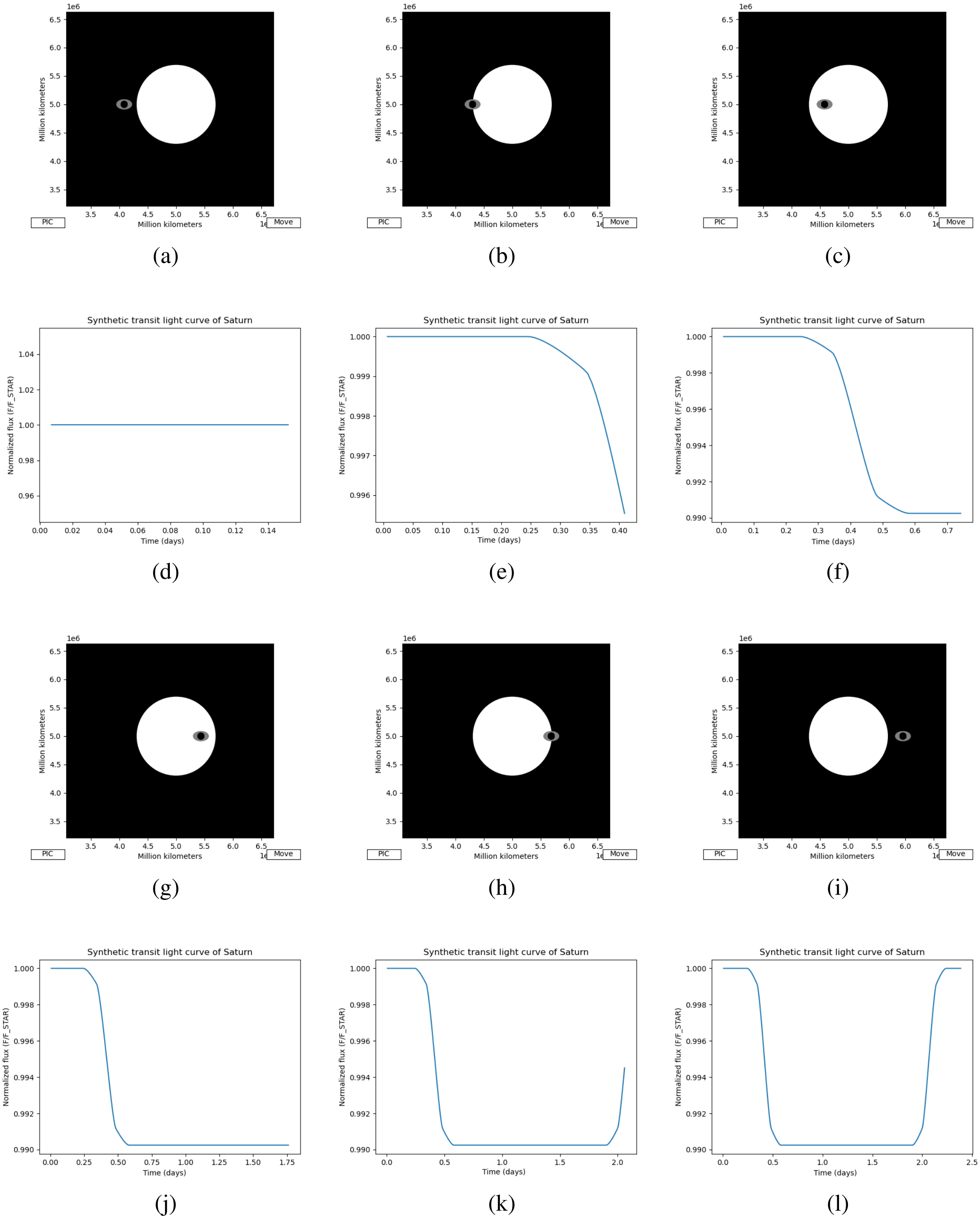


Figure 11: Transit steps [$\tau = 0.1, i_R = 50°$] and their correspondence to the light curve.

4 The Data

Data which is used in this thesis is from the public domain. Even so, it is often not easy to find the respective servers and access the data from there. As mentioned earlier in this thesis we are using the data from Kepler missions. Reason for that is their easier availability compared to other missions as WASP and HATNet and the most of all the best exo-ring candidates are found from Kepler missions by today. A huge reason for that is simply the fact that Kepler missions have found the majority of the all exoplanets known today with a large variety in orbital distance, planet's size and host star's size [2].

4.1 Kepler Space Telescope

A huge improvement in extrasolar planet research occured back in 2009 with the launch of the Kepler space telescope. [21] Kepler satellite was designed to discover exoplanets by transit method. [7] During its prime mission which lasted almost four years Kepler investigated 150 000 main sequence stars. The reason for that was the goal to find Earth-size and smaller planets in habitable zones. In 2014 Kepler's mission 2 (shorter K2) began which lasted also around four years until NASA retired the Kepler telescope in October 2018. [8] Kepler did discover totally on its missions 2745 confirmed extrasolar planets which from 2348 planets were found on its first phase and 397 planets during its K2 phase. [9] The light curve data from Kepler missions is openly available online. [12]

There are also a many other campaigns for exoplanet search as CoRoT (transit method) [33], TESS (transit)[34], GAIA (astrometry)[35], CHEOPS (transit)[36], WASP (transit)[37], OGLE (gravitational microlensing and transit)[38], HARPS (radial-velocity)[39] and upcoming campaigns using the JWST (transit)[40]. However the best exoplanet candidates for having rings are found with Kepler missions. Later in the thesis candidate selection criterions are explained with more details but if we set reasonable limits for a planet density, a semi-major axis and radius the only suitable candidates are from Kepler missions. Therefore this thesis contains candidates only from Kepler missions to find clues of rings around exoplanets.

4.2 Kepler Data

The Kepler data contains a huge disparity between different exoplanets on all attributes. For example orbital periods do vary between over 1000 days to only one third of a day and planet masses from 0.04 Earth masses to tens of thousands of Earth masses. [2] That gives a large variety of planetary systems which could also contain ringed planets.

The light curves which are used in this thesis for Kepler targets are found from the Exomast search service [12]. There you can search exoplanets you want to examine with their name (for example "Kepler-342 c") and then comprehensive web page opens where it is possible to see all available info of the host star, the planet itself and the most important feature for this thesis, it is possible to download the Kepler light curve for that target. In some Kepler targets the light curve files can not be downloaded from Exomast service but Kepler Data Search & Retrieval [50] did cover that.

The light curve data from Kepler is in ".fits" format. It contains much information not needed for plotting the light curve and therefore it is necessary to take a couple of extra steps that the light curve could be plotted. However in practice of astronomy they contain a lot of information which can easily be used for multiple purposes and therefore it is the most used format for astronomical purposes to have data in that format.

Otherwise than in the main Kepler mission data, all confirmed planets from K2 mission are planets with orbital period maximum of 50 days but the size of planets varies a lot from 0.17 Earth masses to thousands of Earth masses. Also the size of a host star varies a lot starting from one fourth of a Sun mass to a couple of Sun masses. [2] Therefore there are also many different circumstances for planets and precise examination is needed on every ring-candidate.

The light curve data from the K2 mission is in a more simple format. The data itself can be found from K2SFF Light Curves Search [13] by entering a target's name, for example "K2-11 b". The light curve data from K2 mission there is already normalized for flux and downloadable in ".csv" format which is simpler than data in ".fits" format because there is no additional information in K2's corrected light curve data, only time-flux data point pairs.

4.3 Filtering Data and Selected Exo-Ring Candidates

To find hints for rings around exoplanets it is not possible and reasonable to go through every confirmed planet. That is why we need to filter which planets we examine and which ones not and there we will follow the so called "Zuluaga" method [4]. The main thing on that method is to find extrasolar planets with abnormally low densities. That is because in transit method the planet's radius is measured from the depth of the transit where deeper transit pattern is proportional to a bigger planet. In other words a planet with greater radius does block more light from its host star than a planet with smaller radius. It causes the star's apparent flux to decrease temporarily. The size of the planet can be derived from the following equation [11]

$$R_p = R_\star \sqrt{b}, \tag{6}$$

where R_p is the radius of the planet, $R_\star$ is the radius of the host star and b is the transit depth. In figure 12 we can see how transit depth would change if we observed from the other star system Jupiter, Saturn and Neptune sized planets to transit the Sun in the Saturn's mean orbital distance. We can see that the size of the planet does affect a lot to the depth of transit.

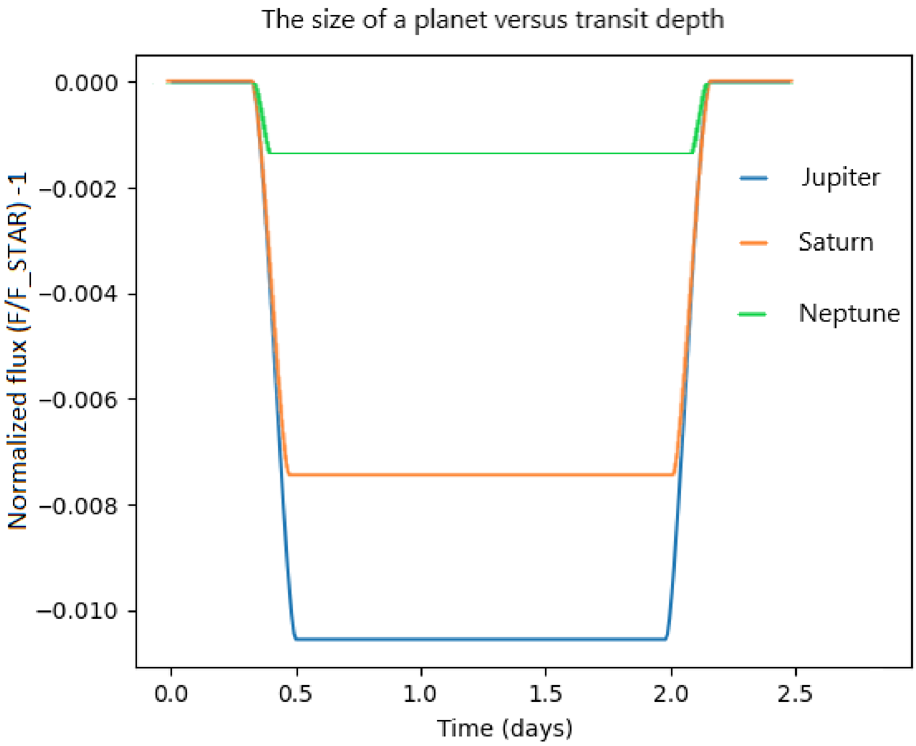


Figure 12: SLCs of Jupiter, Saturn and Neptune without rings. Radii of 71 492 km, 60 268 km and 24 764 km, respectively. Radius of the host star (Sun) used is 696 000 km.

As mentioned earlier if a planet has rings around it, it would leave significantly deeper transit pattern than a same size planet without rings. When the planet's mass is known it is possible to determine the density of the planet and therefore a planet with rings would be determined a much less dense than a planet without rings because its radius is false determined.

Planet's density can not be determined only with a transit method but it needs other methods such as Radial Velocity method too. To be able to measure the mass of the planet one need to calculate the distance from our Solar system to the planet's host star. When the distance is measured the apparent luminosity of the star will be measured, because luminosity is straightly related to mass for given star type. When we know the mass of the star, we can use Kepler's Third Law to measure orbital radius of the planet as follows.

$$P^2 = \Big(\frac{4\pi^2}{G(M_\star + m_{planet})}\Big)a^3 \approx P^2 = \Big(\frac{4\pi^2}{GM_\star}\Big)a^3 \rightarrow a = \sqrt[3]{\frac{P^2GM_\star}{4\pi^2}} \tag{7}$$

where a is the mean orbital radius, P is the orbital period of the planet which can be obtained for example from time difference between transits of the same planet, G is the gravitational constant and $M_\star$ is the mass of the host star. m_{planet} can be ignored at this point because it is a way smaller than the mass of the star. After the orbital distance has been measured the mass of the planet can be obtained by using Newton's Law of Gravitation as follows.

$$F_g = G\frac{M\star m_{planet}}{a^2} \rightarrow m_{planet} = \frac{F_g a^2}{GM_\star} \tag{8}$$

where m_{planet} is the mass of the planet, $M\star$ the mass of the star, a the orbital distance measured earlier, G the gravitational constant and F_g is the gravitational force between the host star and the planet which can be determined for example from the Doppler shift measured by using radial velocity method. [66]

With further examination from the NASA Exoplanet Archive [2] we can notice that the majority of Kepler detections have been observed with other methods which have allowed to determine for example densities. For every exoplanet dealt in this thesis the mass has been measured with other methods and therefore densities can be measured for those planets.

We can take a look at the Solar system giant planets' densities that we can have some hint of how small densities we should be looking for.

Table 2: **Giant planets' densities**

Planet	Density
Jupiter	1,330 $g \cdot cm^{-3}$
Saturn	0,700 $g \cdot cm^{-3}$
Uranus	1,270 $g \cdot cm^{-3}$
Neptune	1,638 $g \cdot cm^{-3}$

So regarding the densities on table 2 above [14] we can assume that regular gas giant's density can vary quite a lot. However it gives us some clue that the obtained density of the planet should not be over 2 grams per cubic centimeter if we wanted to have the best possibilities to find rings around it. That is because if that planet actually had rings around it, its real density would be significantly larger and therefore its composition would not be mostly gas. After all we are looking for giant gas planets because with current knowledge they are the most probable planets to have rings. In radial velocity method star's movement due gravity is observed by measuring its wavelength changes in time. Changes are harder to detect if star-planet size difference is huge and vice versa. Also most of the exoplanets found at the time are gas giants which orbit their host stars with small semi-major axes. It means that these planets have really high surface temperature. Principles of thermophysics show that substance extends when temperature increases and therefore density decreases.

Other filtering actions which we can and should consider are the following:

1. Check for the planet's semi-major axis. If it is short (e.g. < 0.5 AU) there is smaller possibility for a planet to have rings than with greater distance. (equation 1)[48]

2. Check for size of a host star. If a planet orbits its host star close (e.g. < 0.5 AU) and the star itself is huge there apparently should not be a chance for rings existence. (eq. 1)

3. Check for the size of a planet itself. If a planet's size is small (e.g. $< 2R_{\oplus}$) with really high chance it would not have rings. That is because heavier and larger planets have obviously larger gravitational force and therefore the debris for rings is generated more easily. The reason for that is found from strength of the material and the stress it can support. When gravitational force grows up, the stress towards orbiting material grows up and if the

strength of the material is not big enough, the material breaks apart. [67] Usually the size of a planet corresponds to its composition and therefore to its density. That is why this point should not be needed to consider often.

With these points in addition to main criterion, density of a planet, we should be able to obtain the best candidates among all confirmed exoplanets for having rings.

Ring candidates were filtered using the Nasa Exoplanet Archive [2]. There it was possible to filter all confirmed exoplanets with any parameters wanted and finally the following ones were used:

1. Density must be measured and < $0.7g/cm^3$
2. Semi-major axis > $0.13AU$ for K2 candidates
3. Semi-major axis > $0.50AU$ for Kepler candidates
4. Radius of planet > $3.0R_\oplus$

A density < $0.7g/cm^3$ was selected because of our main criterion for selection is that the planet would be underdense. We have presented earlier the densities of gas giants in our Solar system and as seen, Saturn's density is 0.7 g/cm^3. Saturn is the ringed planet of our Solar system, has the lowest density of all Jovian planets and therefore we will use its density as the upper limit for exo-ring candidates.

For Kepler candidates the bigger semi-major axis limit was used because it limited the amount of exo-ring candidates. Otherwise this thesis would have multiple times more candidates than it contains now. Also with increasing semi-major axis the appearance of the ring system is more probable. With these filtering methods the following exoplanets have been selected for further examination. Like stated before K2 mission did focus on exoplanets with shorter orbital periods and therefore there are not that many candidates worth considering in the K2 data. Instead Kepler's mission one data includes much better candidates for having rings because those planets orbit their host stars with significantly larger semi-major axes' and therefore have better chances to have rings.

After applying criteria presented here we get a list of exo-ring candidates shown in table 3. With selected criteria we found three other candidates (Kepler-47 c, Kepler-34 b and Kepler-35 b), but they are not examined further in this thesis because each of them orbited a double-star system and the SLC program is developed for only single star systems.

Table 3: **Ringed planet candidates**

Name	$\rho\ (g/cm^3)$	$a(AU)$	$R_{planet}\ (R_\oplus)$	$R_{star}\ (R_\odot)$	Orb. Per. (days)
Kepler-51 d	0.046	0.509	9.7	0.940	130.194
Kepler-87 c	0.152	0.676	6.14	1.82	191.232
Kepler-167 e	0.386	1.890	10.15	0.726	1071.232
Kepler-108 c	0.511	0.721	8.18	2.192	190.323
Kepler-111 c	0.593	0.761	7.30	1.157	224.785
K2-24 c	0.20	0.247	7.5	1.16	42.3391
K2-280 b	0.555	0.1488	7.67	1.28	19.895
K2-11 b	0.567	0.2257	7.550	5.15	39.938

5 Comparison of the Model to a Choice of Kepler Lightcurves

Before starting to examine the light curves of ring candidates the expectations for finding clues about rings is not high. All K2 data contains only data for exoplanets which orbit really close to their host star. The selected K2 candidates among K2 data were the best candidates of them all. In general the transit method is not the best method to find exoplanets with large semi-major axes because the time between transits grow up and statistically the possibility of finding transiting planet near its host star is significantly bigger than finding a planet transiting with a long semi-major axis. The probability to detect the planet transiting can be achieved from next formula [68]

$$P_{transit} = \frac{R_\star}{a} \tag{9}$$

where $P_{transit}$ is the probability of transit, $R_\star$ is the radius of the host star and a is the mean orbital distance of the transiting planet. For example in our own Solar system the probability of transit for Mercury is 1.19% when for Saturn it is only 0.049%. That is one major reason why the current data which is available does not have more than a handful of exoplanets whose orbital distance is larger than one AU and therefore there are not many candidates available where at least in principle rings could be expected.

All K2 candidates may orbit too close to their host stars which is not a suitable environment for a ring system. Kepler data may have more possibilities for ring systems since some of the planets from Kepler data orbit with greater distances and have better circumstances for stable ring systems. The most promising candidate among the all selected exoplanets seems to be Kepler-167 e. It has the best suited values for all important parameters such as density, size ratio of a planet and a star and orbital distance.

5.1 Kepler-51 d

5.1.1 Parameters of the Planet Inferred from Observations

Kepler-51 d	
Planet density (g/cm^3)	0.046
Orbital period $(days)$	130.194
Orbit semi-major axis (AU)	0.509
Planet radius $(R_\oplus)$	9.7
Stellar radius $(R_\odot)$	0.940
Luminosity $(L_\odot)$	1.039

Kepler light curves are little different from K2 light curves since there are only light curve files for one transit available. It means that we need to make conclusions just based on one transit pattern on every candidate. However because Kepler light curves are in different file format (.fits), they have a parameter which allows light curves to be modelled differently and in this case it means that light curves are more accurate than for K2 data. Unlike for K2 candidates, the flux is "normalized to 0". In practice it means that the normalization of the flux have been made as presented earlier in the thesis, but the scale have been moved so that the normal flux is in 0. In other words, 1 has been subtracted from normalized flux.

This candidate Kepler-51 d is from the currently known exoplanets third least dense existing. [2][55] Its density is really abnormally low which makes it a great candidate for having a ring system according to Zuluaga [4]. In the figure 13 we can see the transit light curve of Kepler-51 d and to make it easier to find features of rings in the transit pattern there is a red line which is the best suited model for the light curve. That model is included already in the .fits files and described as "The transit model calculated for the initial light curve". Signal-to-noise ratio of Kepler-51 is not the best possible and therefore we need to take that into account when we examine the transit pattern more precisely. Low signal-to-noise ratio may hide ring features from the light curve. In the figure 14 (a) we can see that transit pattern more closely. It follows quite nicely the shape of SLC seen in figure 14 (b) and the transit depth is not significantly deeper. With a green line we can see a SLC with slightly bigger optical depth ($\tau = 0.2$) when the synthetic model fits even better with the real light curve. The features of ring transits are not that clearly seen. If Kepler-51 d had a ring system, that would be explained by low S/N-ratio of data which causes lack of ring features or then different ring orientation. Kepler-51 d could not have icy but only rock composed rings because the ice-border according to equation 1 is at 2.75 AU. However Kepler-51 d could possibly have a ring system because of similarities in light curves.

5.1.2 Light Curves

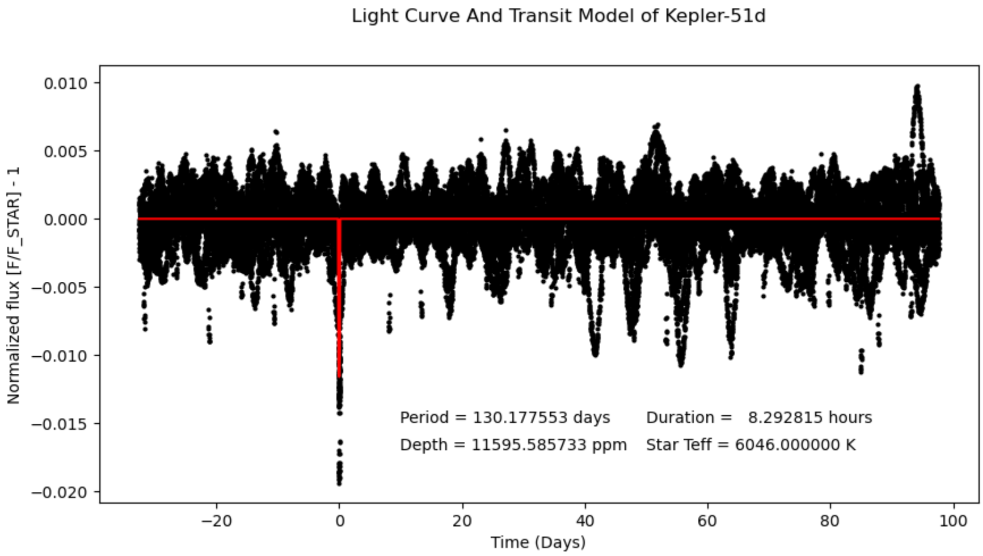
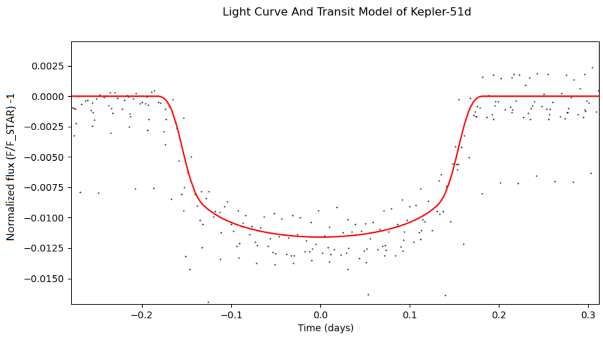
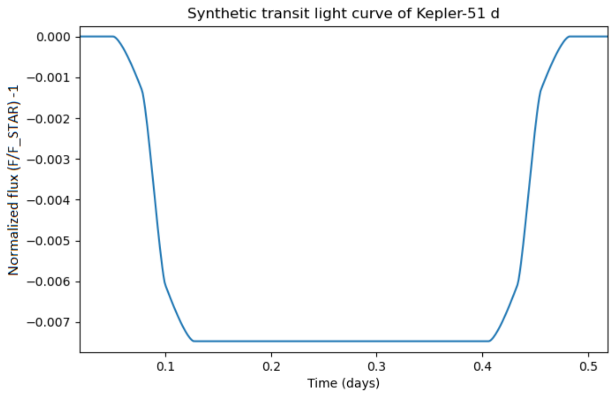


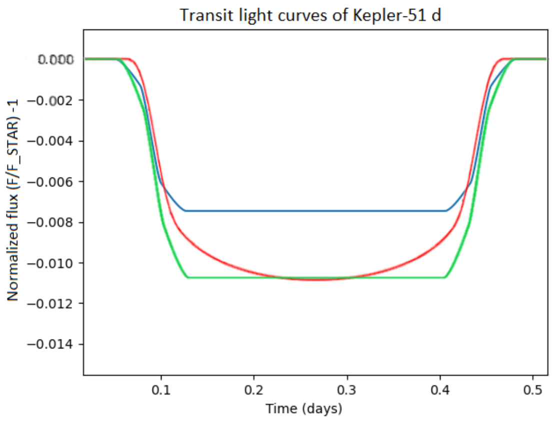
Figure 13: The light curve of Kepler-51 and values for orbital period (DAYS), transit duration (HOURS), transit depth (PPM/parts per million) and host star's effective temperature (KELVINS) obtained from .fits file.



(a)



(b)



(c)

Figure 14: The transit of Kepler-51 d (a) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 40169km$] (b). All light curves overplotted in (c) [Blue line = SLC with original values, red line = real light curve, green line = SLC with $\tau = 0.2$ instead of 0.1].

5.2 Kepler-87 c

5.2.1 Parameters of the Planet Inferred from Observations

Kepler-87 c	
Planet density (g/cm^3)	0.152
Orbital period $(days)$	191.232
Orbit semi-major axis (AU)	0.676
Planet radius $(R_\oplus)$	6.14
Stellar radius $(R_\odot)$	1.82
Luminosity $(L_\odot)$	2.92

Kepler-87 c is a medium size gas planet. Our solar system does not have any this size planets when Uranus and Neptune have around 4 times larger radius than Earth, Saturn has 9 times larger radius and Jupiter about 11 times larger radius. The composition of this kind of light gas giants as Kepler-87 c is not understood well [56] and therefore one thing which could explain abnormally low detected densities would be ring systems. This exoplanet does orbit its host star with the biggest orbital distance what we have examined so far which gives a high expectations.

When taking a look at the light curve of Kepler-87 in figure 15 we can immediately notice that the quality of the light curve is a lot better than with the former candidate. The transit pattern is again modelled with the red line which is seen more precisely in figure 16 (a). Transit pattern of Kepler-87 c is slightly different from Kepler-51 d's with slightly steeper edges. However there are again seen a lot similarities between shapes of the real light curve and the SLC. The transit depth however is about three times as deep in the real light curve which could be explained with a smaller inclination angle. According to Akinsanmi et al. [16] the transit depth in general with $i_R = 0°$ is about twice as deep as with $i_R = 70°$ if $\tau = 0.5$. Also greater optical depth of rings makes transit deeper. Transiting time in SLC is twice as long as in a real light curve but it can be explained for example with shorter transit path in real. Kepler-87 c may have been observed transiting its host star closer to the edges which shortens transit time. We can see an alternate SLC with fixed parameters in figure 16 (c). With green line optical depth τ is changed to 0.5 and inclination angle i_R to 30 °. After changes a SLC becomes nearly identical in shape with the real light curve with only the transit time being still longer. When also the transit time is shortened by 1/3 the duration of transit becomes quite identical also. The composition of rings could not be ice, because ice-border of the Kepler-87 system is at 4.61 AU. However there is a chance that this planet could also have a rock composed ring system.

5.2.2 Light Curves

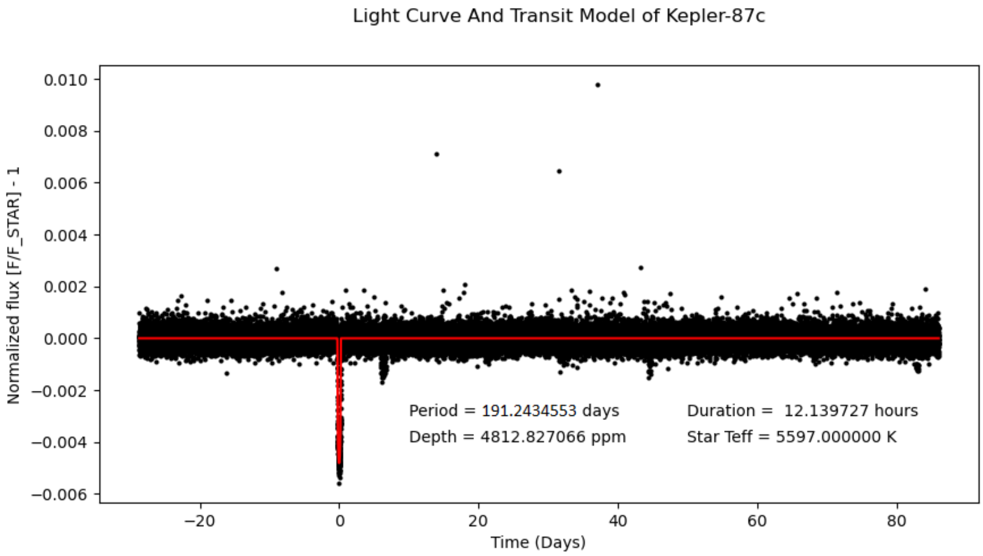
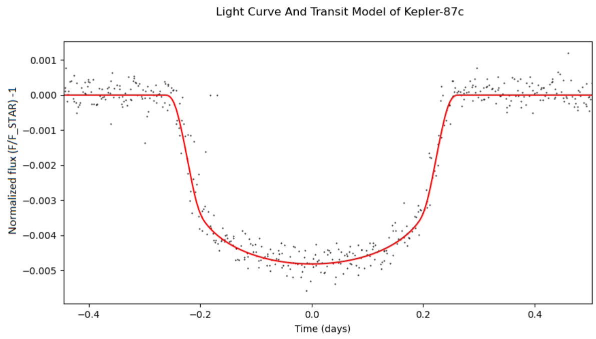
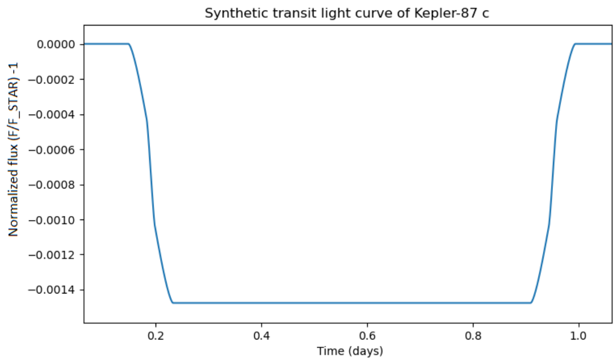


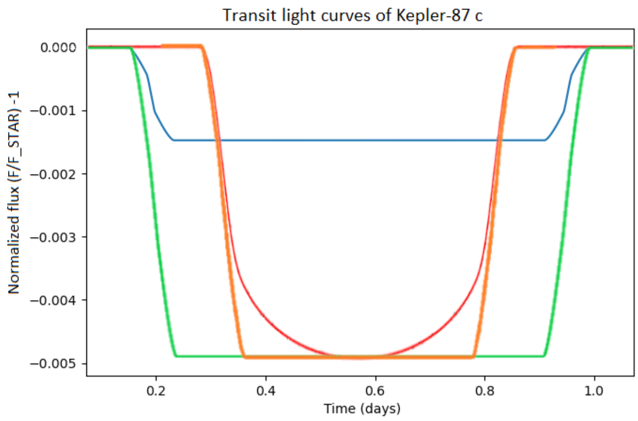
Figure 15: The light curve of Kepler-87 and values for orbital period (DAYS), transit duration (HOURS), transit depth (PPM/parts per million) and host star's effective temperature (KELVINS) obtained from .fits file.



(a)



(b)



(c)

Figure 16: The transit of Kepler-87 c (a) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 25426km$](b). All light curves over-plotted in (c) [Blue line = SLC with original values, red line = real light curve, green line = SLC with changed values $\tau = 0.5$ and $i_R = 30°$, orange line = $\tau = 0.5, i_R = 30°$ and duration of transit 2/3 of the original SLC's transit duration].

5.3 Kepler-167 e

5.3.1 Parameters of the Planet Inferred from Observations

Kepler-167 e	
Planet density (g/cm^3)	0.386
Orbital period $(days)$	1071.232
Orbit semi-major axis (AU)	1.890
Planet radius $(R_\oplus)$	10.15
Stellar radius $(R_\odot)$	0.726
Luminosity $(L_\odot)$	0.27

Kepler-167 e was one of the best candidates of all exoplanets to have rings just by looking at its parameters. Its density is really small, it has almost 2 AU's orbital distance, the size of a planet is between Saturn and Jupiter and its host star is a cool and small K4 type star. [57]

From the figure 17 we can see the light curve of Kepler-167 e and from the first sight data looks quite high-quality. When taking a closer look of the transit pattern in figure 18 we can notice that there are not that many data points than in earlier light curves which makes the light curve model less accurate. Again the transit pattern is not that steep than in some of the earlier targets and there may be actually seen little transit wings marked with red circles. However because the lack of data points it is not accurate to say for sure that those transit wings really appear and when comparing to the SLC we notice that those transit wings should be a way wider than they are in real light curve. In other hand short transit wings could be explained with different ring orientation or smaller ring system. When we compare light curves in figure 18 (c) we can notice that length of the transit is quite identical in both. Also transit depths have not that large difference between each other. Again slightly bigger optical depth could make a such difference which would steep the transit wings also and in that case light curves would be even more identical. Such a change can be seen in figure 18 (c) as a green line. There optical depth τ have been changed slightly, to 0.2. Unlike two former candidates, Kepler-167 e could have icy composed rings since it locates outside of the ice-border (1.40 AU). With these evidences we can also say that Kepler-167 e could have a ring system.

5.3.2 Light Curves

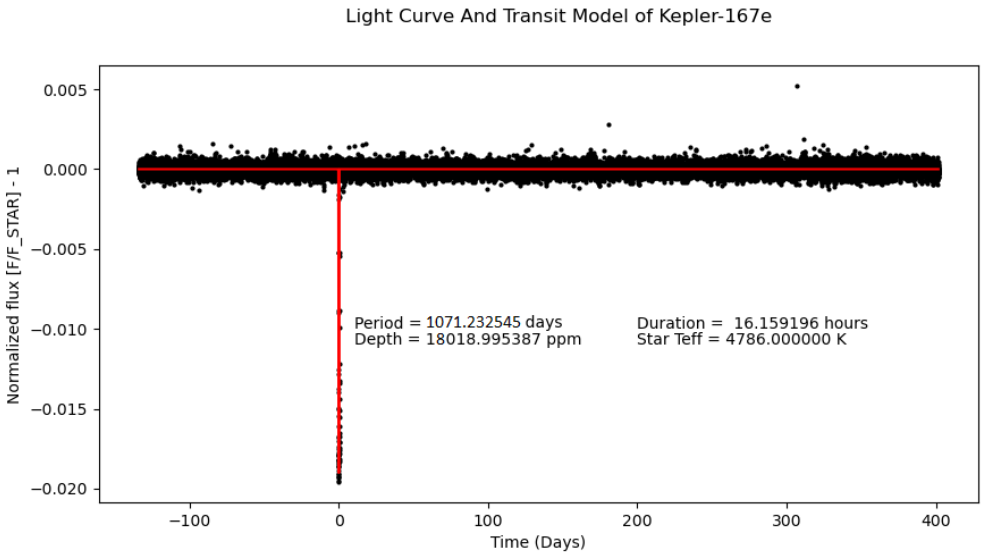
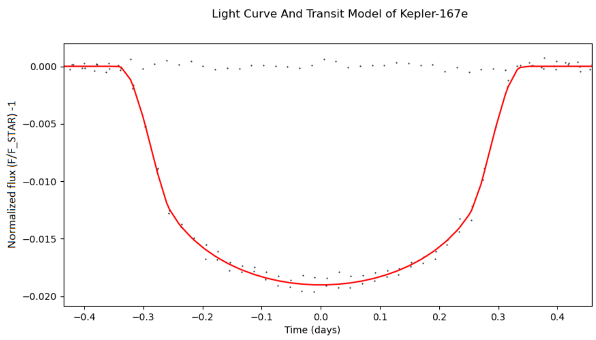
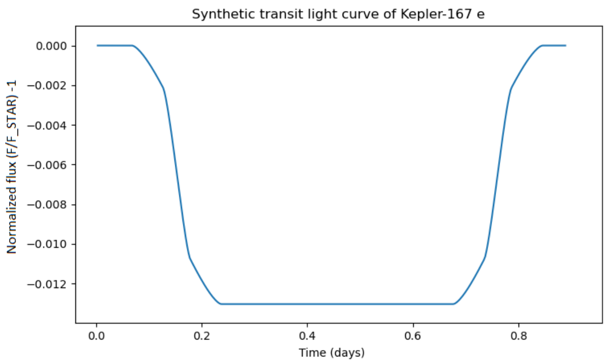


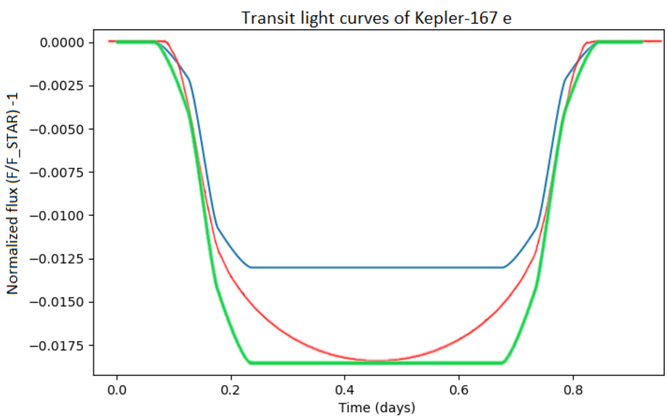
Figure 17: The light curve of Kepler-167 and values for orbital period (DAYS), transit duration (HOURS), transit depth (PPM/parts per million) and host star's effective temperature (KELVINS) obtained from .fits file.



(a)



(b)



(c)

Figure 18: The transit of Kepler-167 e (a) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 42032km$](b). All light curves over-plotted in (c) [blue line = SLC with original values, red line = real light curve, green line = SLC with changed value of $\tau = 0.2$].

5.4 Kepler-108 c

5.4.1 Parameters of the Planet Inferred from Observations

Kepler-108 c	
Planet density (g/cm^3)	0.511
Orbital period $(days)$	190.3235
Orbit semi-major axis (AU)	0.721
Planet radius $(R_\oplus)$	8.18
Stellar radius $(R_\odot)$	2.192
Luminosity $(L_\odot)$	5.06

Kepler-108 c is another same type of exoplanet than many earlier candidate; Its size is around Saturn's size and it orbits its host star with Sun-Venus distance. The most promising parameter is again its density which is quite low.

When examining the light curve in the figure 19 we can notice that it contains multiple dips where the deepest one is Kepler-108 c. We can see a distinct transit more closely in figure 20. The transit pattern is quite steep and there can not be seen that clear features of rings. Though the SLC does not show ring features very clearly too because of the size difference between Kepler-108 c and its host star. If a star is a lot bigger than a transiting planet it causes transit features to be harder to detect. But actually the shapes of both light curves have a lot of similarities. Again the transiting time is shorter in an observed light curve but it can be explained with shorter transiting path like mentioned earlier. Also there is a small difference between transit depth but that can be explained with slightly different ring parameters also as shown earlier. With a green line in figure 20 (c) a SLC is tried to fit with the real light curve by changing $\tau = 0.18$ and shortening the transit time with 40%. After the fit SLC with changed values does fit with the real light curve really well. As to the composition of the supposed rings they can not be consisted of any icy material since according to equation 1 the ice-border goes at 6.07 AU's. However this planet could have a rock composed ring system as well.

5.4.2 Light Curves

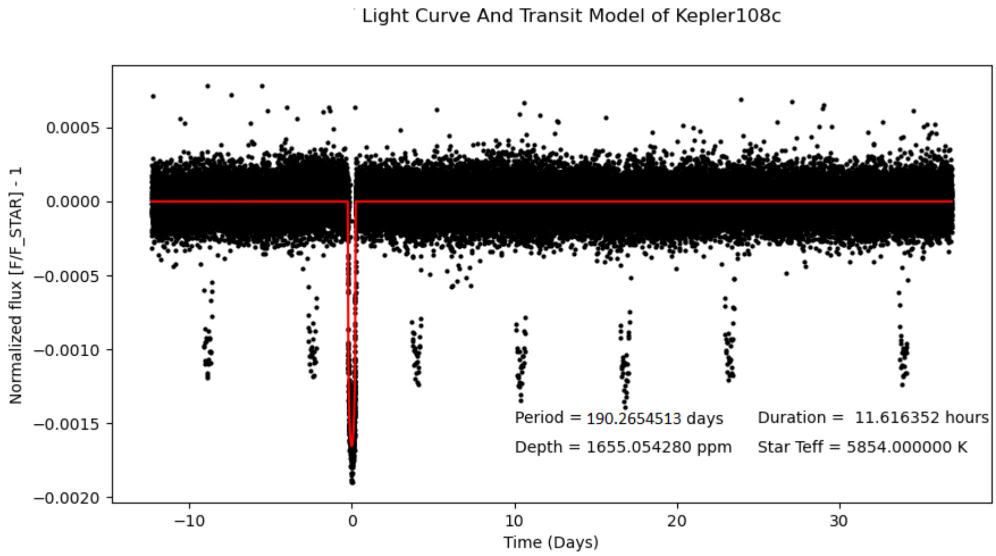
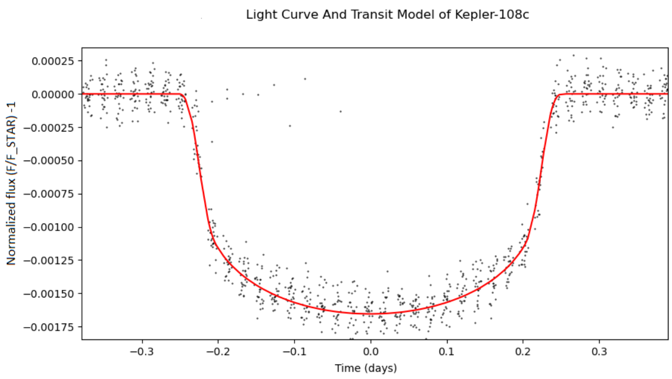
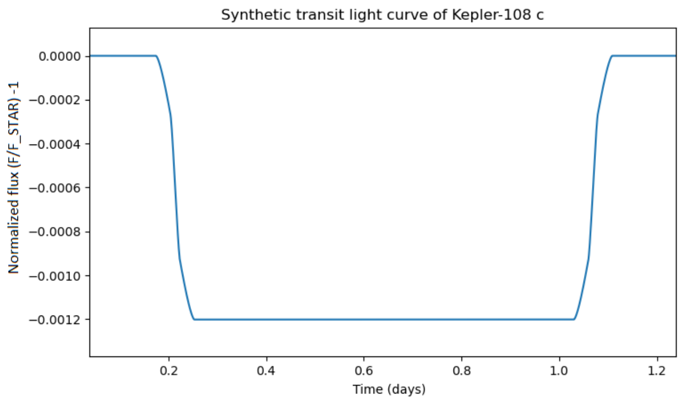


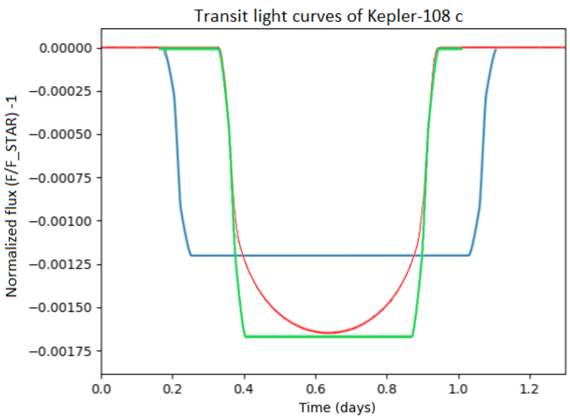
Figure 19: The light curve of Kepler-108 and values for orbital period (DAYS), transit duration (HOURS), transit depth (PPM/parts per million) and host star's effective temperature (KELVINS) obtained from .fits file.



(a)



(b)



(c)

Figure 20: The transit of Kepler-108 c (a) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 33874km$](b). All light curves overplotted in (c) [blue line = SLC with original values, red line = real light curve, green line = SLC with changed value of $\tau = 0.18$ and 40% shorter transit time].

5.5 Kepler-111 c

5.5.1 Parameters of the Planet Inferred from Observations

Kepler-111 c	
Planet density (g/cm^3)	0.593
Orbital period $(days)$	224.785
Orbit semi-major axis (AU)	0.761
Planet radius $(R_\oplus)$	7.30
Stellar radius $(R_\odot)$	1.157
Luminosity $(L_\odot)$	1.506

Kepler-111 c is almost like an identical twin with the former Kepler-108 c with only one big difference; the size of its host star. Therefore this exoplanet has slightly better circumstances for having rings since its host star is twice smaller.

The light curve itself in the figure 21 has not so many data points as some earlier Kepler candidates but we can still see a pretty clear transit on it. When examining the transit pattern more closely in the figure 22 we can notice that it is almost identical in shape with the former transit pattern of Kepler-108 c. The edges of the light curve are not that steep than with Kepler-108 c and the transiting time matches nearly with the SLC in figure 22 (b) as well as transit depth. For example with minor change in optical depth $\tau = 0.1 \rightarrow 0.07$ and by shortening the transit time with 25% we get those light curves almost identical as seen in figure 22 (c). On Kepler-111 system the host star's luminosity is not very high, but ice-border locates at 3.31 AU and therefore icy rings can not exist but only rocky composed. Again, this planet could still also have a ring system on it.

5.5.2 Light Curves

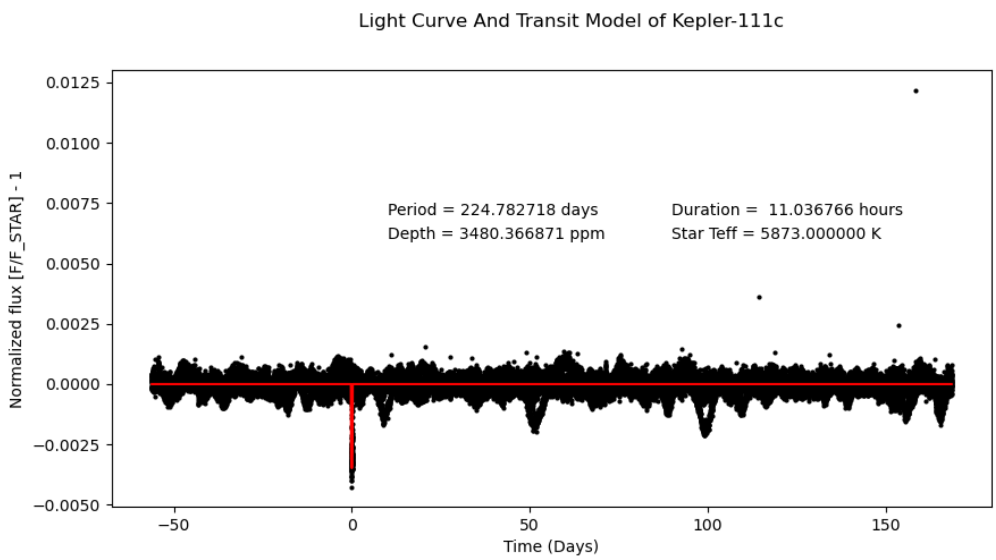
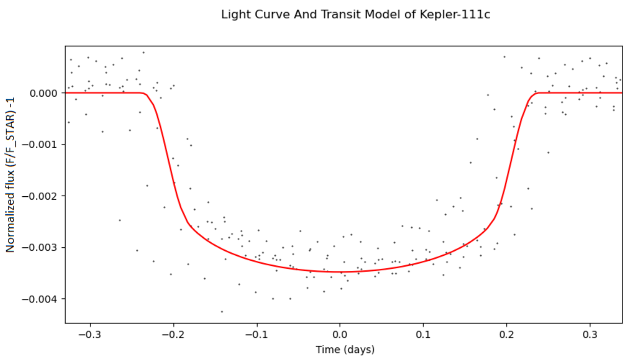
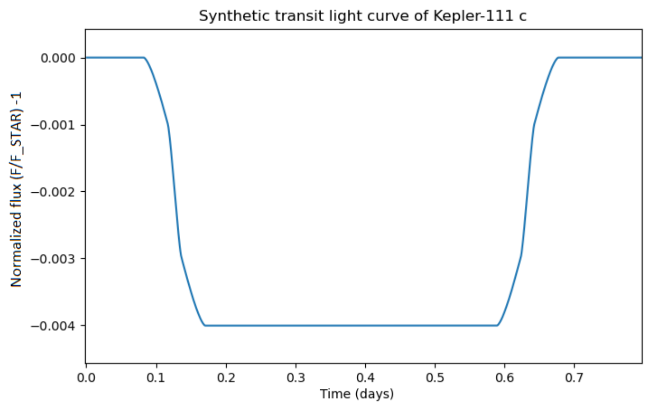


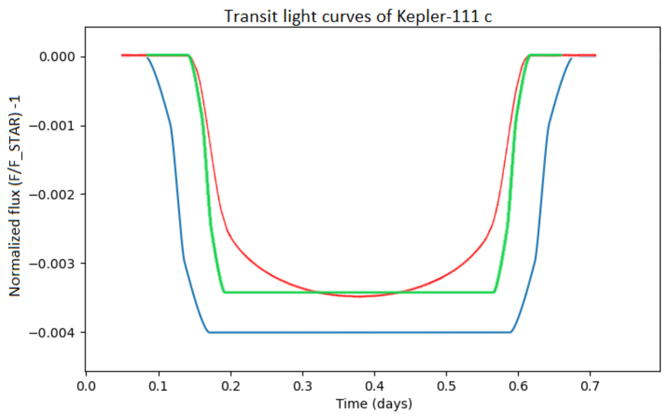
Figure 21: The light curve of Kepler-111 and values for orbital period (DAYS), transit duration (HOURS), transit depth (PPM/parts per million) and host star's effective temperature (KELVINS) obtained from .fits file.



(a)



(b)



(c)

Figure 22: The transit of Kepler-111 c (a) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 30230km$] (b). All light curves over-plotted in (c) [blue line = SLC with original values, red line = real light curve, green line = SLC with changed value of $\tau = 0.07$ and 25% shorter transit time].

5.6 K2-24 c

5.6.1 Parameters of the Planet Inferred from Observations

K2-24 c	
Planet density (g/cm^3)	0.20
Orbital period $(days)$	42.3391
Orbit semi-major axis (AU)	0.247
Planet radius $(R_\oplus)$	7.5
Stellar radius $(R_\odot)$	1.16
Luminosity $(L_\odot)$	1.208

The first exoplanet from K2 mission which will be examined has a really low density. It also probably is the so-called "hot Jupiter", which is a generally used term for gas giant exoplanets which orbit their host star really close and therefore their surface temperatures are really high. Every candidate K2 data offers for this thesis will be "hot Jupiter". However we will examine these since their densities are abnormally low and therefore they could in theory have rings around them.

As seen in the figure 23 the light curve of K2-24 has actually transits of two different planets. The deeper ones near days 2080 and 2125 are the transits of K2-24 c which will be examined more closely. In the figure 24 (a) we can see the transit pattern more closely around day 2080 and there we can actually see some kind of transit wings which are the main feature of the transit pattern if a planet has rings around it according to Zuluaga [4]. However the wings are caused by bad signal-to-noise ratio and lack of the data points. We can double-check that by taking a look of the second transit more closely. In figure 24 (b) we can see the second transit. There we actually can not see any signs of those transit wings but only steep transit patterns. Finally we see by comparing real transits to the synthetic one in figure 24 (d) that real transits match otherwise quite well with synthetic transit but ring features can not be precisely seen. Ice-border of K2-24 is at 2.97 AU's. Definite conclusions about rings are not possible to do because of lack of data points.

5.6.2 Light Curves

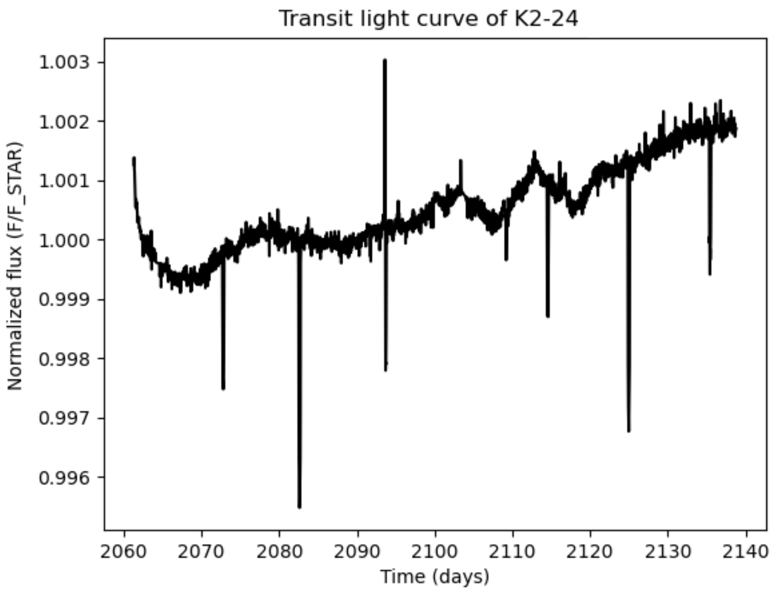
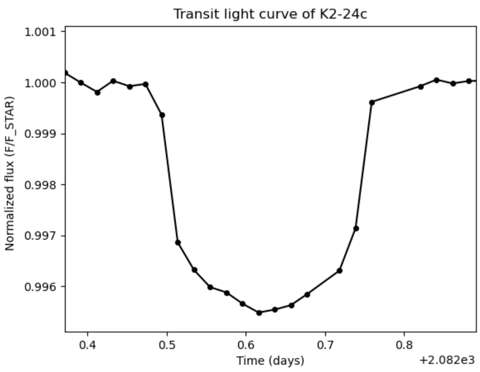
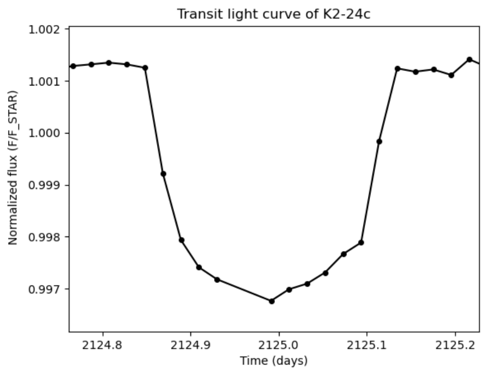


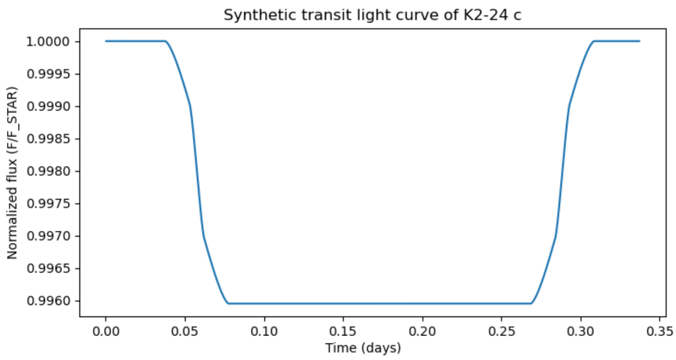
Figure 23: The light curve of K2-24. Transits of K2-24 c in about days 2083 and 2125



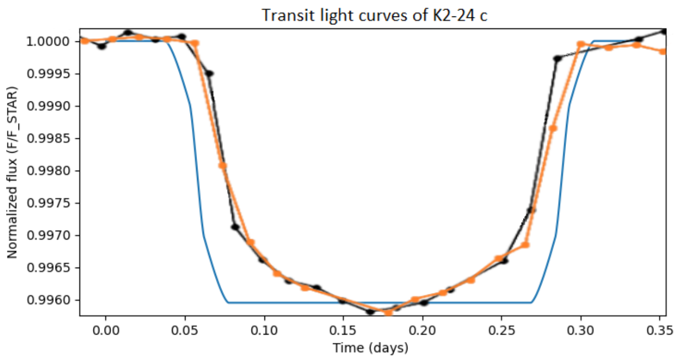
(a)



(b)



(c)



(d)

Figure 24: Two individual transits of K2-24 c (a,b) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 31058km$] (c). All light curves overplotted in (d) [blue line = SLC with original values, black line = first transit, orange line = second transit].

5.7 K2-280 b

5.7.1 Parameters of the Planet Inferred from Observations

K2-280 b	
Planet density (g/cm^3)	0.555
Orbital period $(days)$	19.895
Orbit semi-major axis (AU)	0.1488
Planet radius $(R_\oplus)$	7.67
Stellar radius $(R_\odot)$	1.28
Luminosity $(L_\odot)$	1.597

K2-280 b is also a hot giant gas planet since it orbits its host star very close and its size is somewhat bigger than Uranus or Neptune. When we examine its light curve on figure 25 we see three transits of the same planet there. One transit between the first and second dips in the lightcurve is for some reason missing. There can be many reasons for that for example interplanetary dust which have affected the light curve that way.

With a more closer look at the three transits in figure 26 we can not see any kind of clues about rings from the shape of transit patterns. The second transit in figure 26 (b) has a prominent flux peak in the middle of transit. It can for example be caused when a planet transits over a big star spot. Star spots are dimmer regions in stars and therefore can cause a peak like that when transited. The edges of the transit pattern are really steep which is the case with exoplanets without rings and we can also see that if this exoplanet had Saturn-size rings there would be clear signs of them in the light curve. But again because of lack of data points we can not do any definite conclusions about rings.

5.7.2 Light Curves

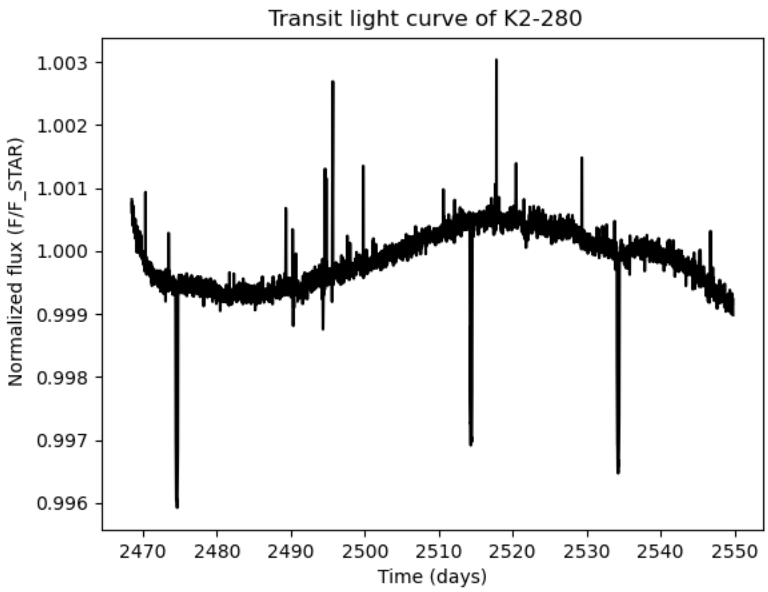
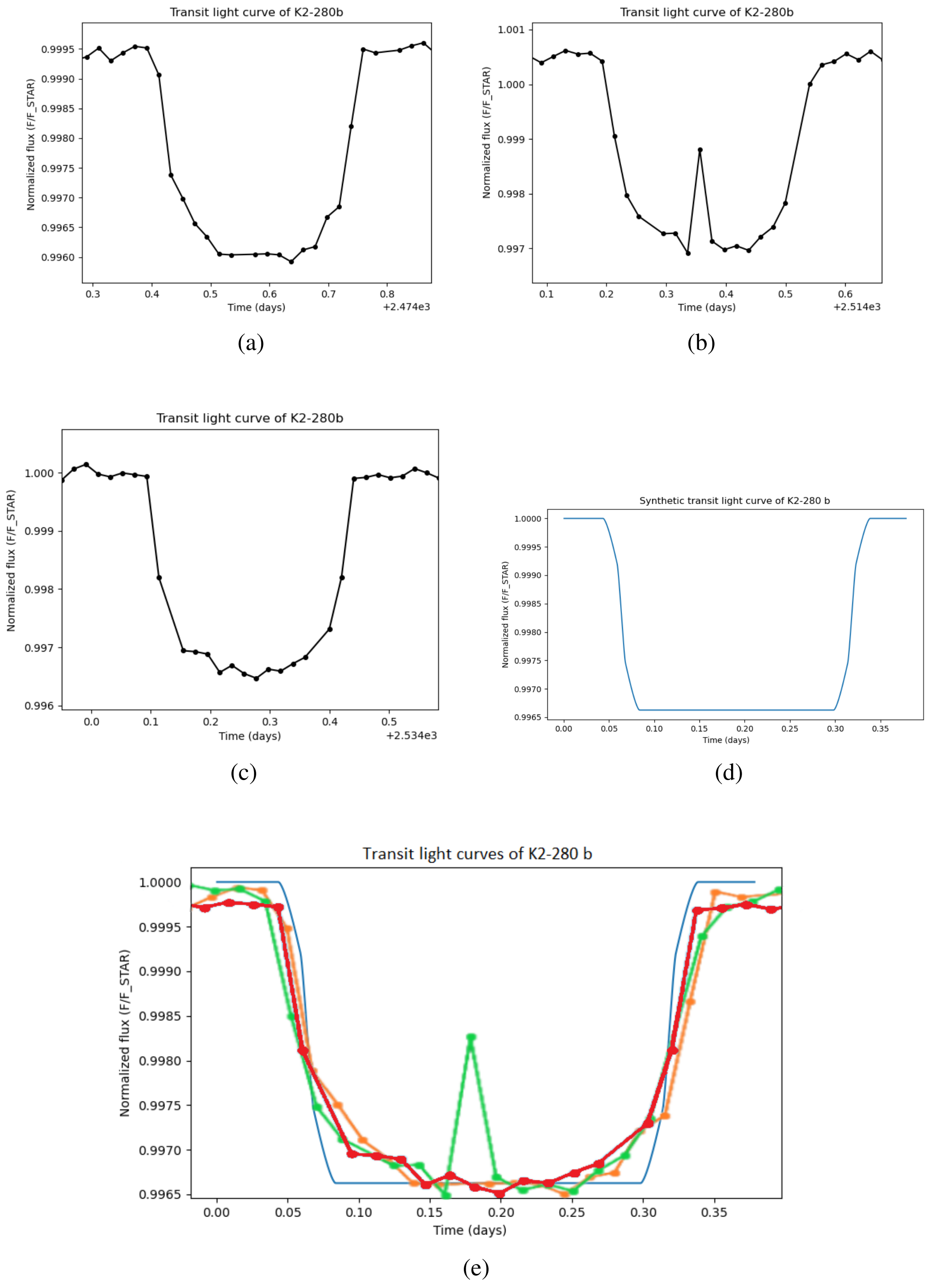


Figure 25: The light curve of K2-280. Visible transits of K2-280 b in about days 2475, 2514 and 2534



(a) (b) (c) (d) (e)

Figure 26: Three individual transits of K2-280 b (a,b,c) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 31762km$] (d). All light curves overplotted in (e) [blue line = SLC with original values, orange line = first transit, green line = second transit, red line = third transit].

5.8 K2-11 b

5.8.1 Parameters of the Planet Inferred from Observations

K2-11 b	
Planet density (g/cm^3)	0.567
Orbital period $(days)$	39.938
Orbit semi-major axis (AU)	0.2257
Planet radius $(R_\oplus)$	7.550
Stellar radius $(R_\odot)$	5.15
Luminosity $(L_\odot)$	20.717

The next and final candidate is almost identical in size and density with former candidate K2-280 b, but there is one huge difference; the host star. This candidate's host star has multiple times greater radius than any other candidate's in this thesis, but some variety between objects is great to have and otherwise K2-11 b is one of the best candidates among K2 objects.

In the figure 27 is seen the light curve of K2-11 b. We can not actually see the transits that clearly without further examination. Transits of K2-11 b are marked with red arrows. The reason for such weak transit patterns is in the size of the host star. When star's size grows that high, the transit depth is small [equation. 6]. From next figure 28 we can see more closely the transit patterns. The signal-to-noise ratio of transits is really poor and therefore there is not any possibility to make any conclusions about K2-11 b's possible rings. K2-11 host star is really bright compared to the Sun and according to equation 1 ice-border in the system goes as far as in 12.29 AU's. It is very probable that a planet in the system like that with small orbital radius can not have any kind of ring system. When comparing the length of transit from real light curves to SLC we can notice that the difference is huge. The most probable reason is found from the data; Because of a bad quality whole transits are not seen properly but only parts of it. We can also clearly see from SLC that transit depth is low for that size-difference between planet and star and therefore it would require very advanced technology to identify the features of transit patterns.

5.8.2 Light Curves

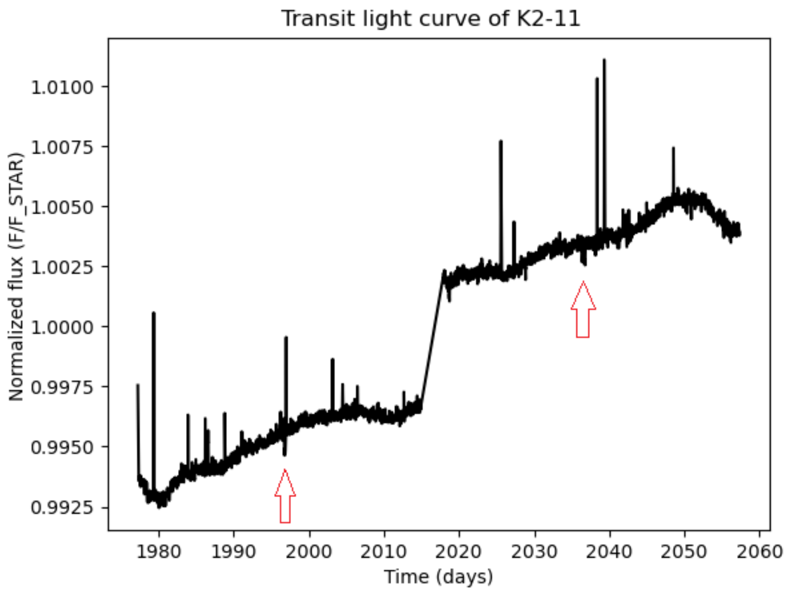


Figure 27: The light curve of K2-11. Transits of K2-11 b are shown with red arrows.

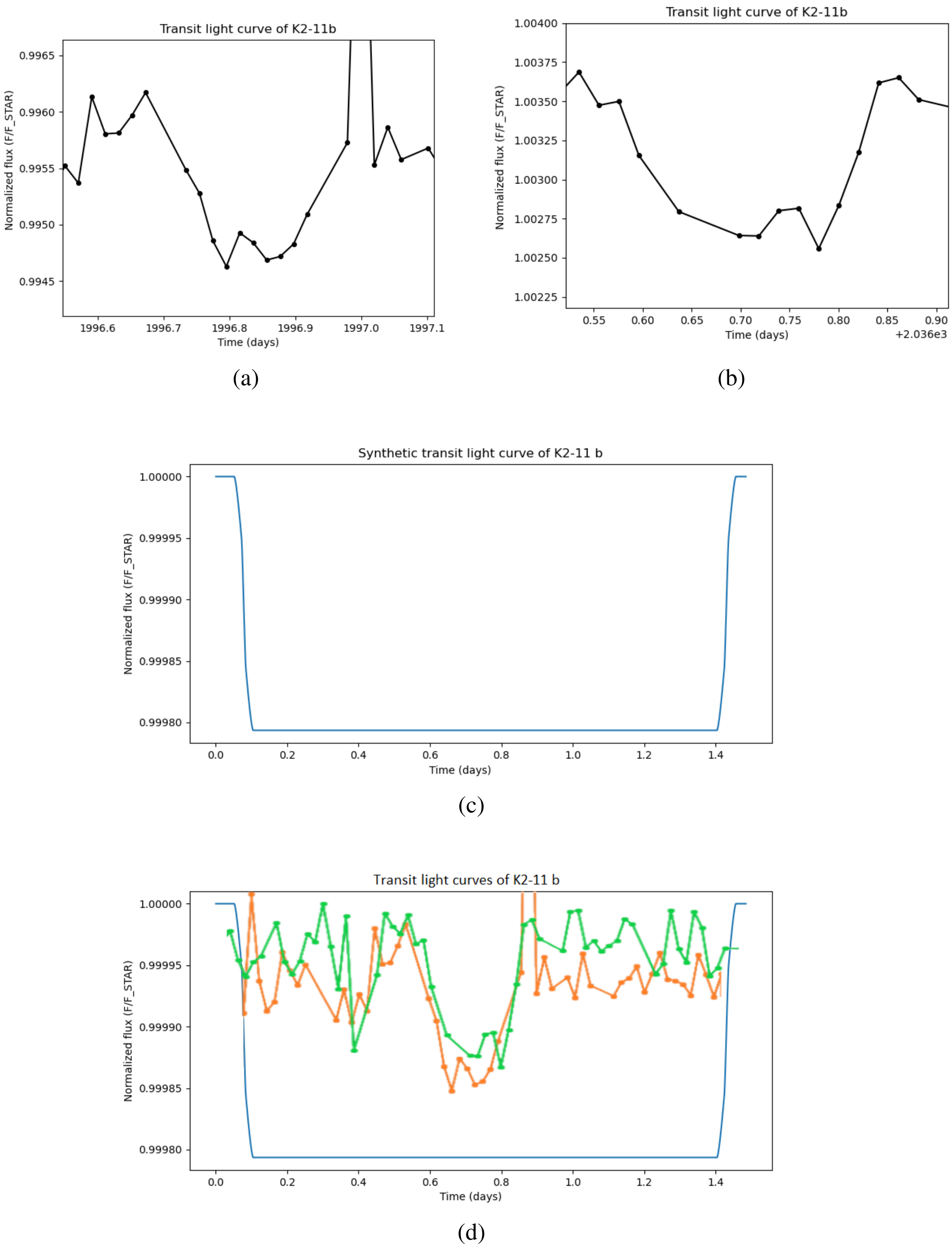


Figure 28: Two individual transits of K2-11 b (a,b) and SLC [$\tau = 0.1, i_R = 50°, R_{irings} = 7000km, R_{orings} = 140000km, R_{planet} = 31265km$] (c). All light curves overplotted in (d) [blue line = SLC with original values, orange line = first transit, green line = second transit].

6 Conclusion

Before examining the light curve data of selected exoplanets the assumption was that any K2 data's candidate would not probably have good enough circumstances for a ring system. That may be right because we did not find any clear ring features from the light curves. In some transits transit wings were appearing but since K2 data did have minute data points which means that the shape of transit patterns were not that accurate, we can not trust that those features in light curves do really exist. Because of the quality of data we can not also make any definite conclusions in general.

Kepler mission one data instead was interesting in many ways. It offered exoplanets with various parameters and circumstances. Every single Kepler light curve did have a lot of data points compared to K2 light curves and therefore it did not play a role in accuracy. However the lack and/or distribution of data points did still play a minor role with some candidates and therefore the shape of transit patterns were not that accurate in every case. We were able to try to fit SLC's with the real light curves on Kepler candidates but it was not possible to do for K2 candidates because of their lack of data points. For all Kepler candidates we were able to imitate quite well their real light curves by changing values of optical depth τ and/or inclination angle i_R and/or the duration of the transit.

Generally among all the candidates and their light curves we discover that if exoplanets had a ring systems they could vary a lot in size and composition. If a planet had a regular ring system, it could still be impossible to detect that from the light curve for sure. Attributes of the ring system as optical depth can vary in the way that we can not separate ring's transit from the planet's transit in the light curve. We notice that if we wanted to detect exoplanets with ring systems we would need a really high-quality data available to see features of rings in transits. From the list of candidates chosen for examination only Kepler-167 e could have ring system with ice composition while all other candidates can have only rocky composed rings because of their host stars' high luminosities.

For summary here are the results of examining light curves in table 4.

Table 4: **Results**

Ring features may exist
Kepler-51 d, Kepler-87 c, Kepler-108 c, Kepler-111 c, Kepler-167 e
Ring features not found
Impossible to classify
K2-24 c, K2-280 b, K2-11 b

The research made for this thesis can be improved in future researches. There are many ways to increase the credibility of results. For example optics of telescopes can be improved which would unveil new exoplanets and increase the quality of current data. One could go through planets with higher densities too. Also the SLC program could be improved so that it could handle more various cases with all possible ring orientations, transit paths, multi-region rings with different compositions and take account of self-rotation of a planet system and a star plus changes in stellar disk (starspots [28]). Obviously with Kepler candidates an easier availability for multiple transits in light curves were also an improvement. In that case we could compare transits of the same planet and see if features of transits are repeated. With research of extrasolar rings we will find fundamentally important attributes of rings and can understand better their evolution.

7 Appendix

7.1 Appendix A

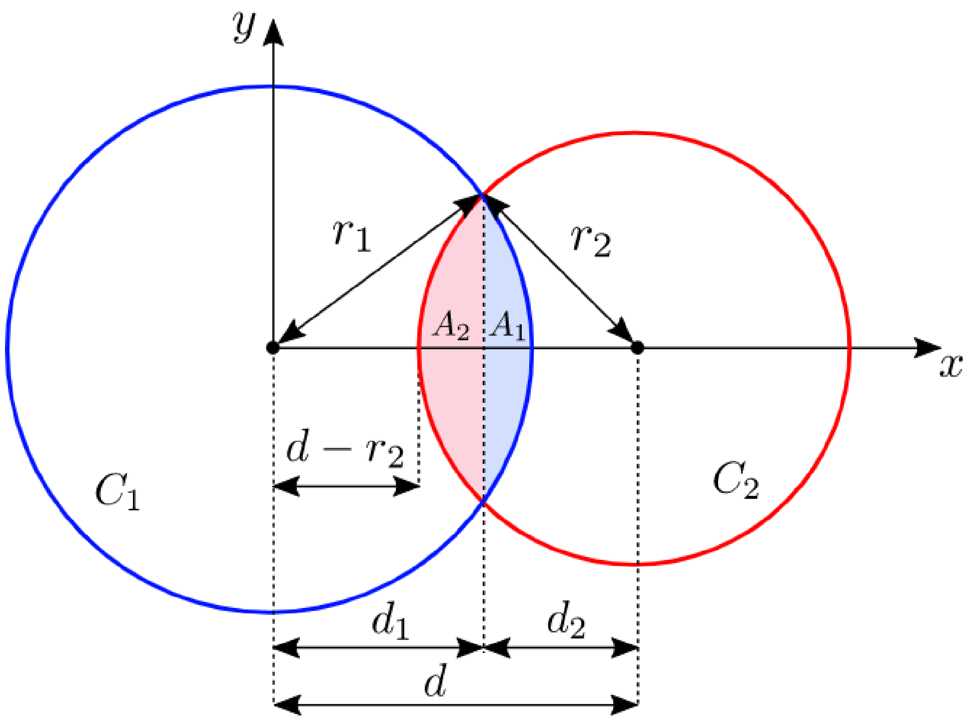


Figure 29: Intersection of two circles [Credit: [15]]

Intersection area of two circles can be obtained by following surface integral. Circles can be described with equations:

$$x^2 + y^2 = r_1^2 \tag{10}$$

$$(x - d)^2 + y^2 = r_2^2 \tag{11}$$

Intersection points are on distance $x = d_1$. Next we need to determine what is d_1. We get

$$r_1^2 - d_1^2 = r_2^2 - (d_1 - d)^2 \rightarrow d_1 = \frac{r_1^2 - r_2^2 + d^2}{2d} \tag{12}$$

The intersection area which is seen as blue and red area in figure 29 will be referred as A_1 and A_2 later on. Therefore we have:

$$A_1 = 2\int_{d_1}^{r_1} \sqrt{r_1^2 - x^2}dx \tag{13}$$

$$A_2 = 2\int_{d-r_2}^{d_1} \sqrt{r_2^2 - (x-d)^2}dx \tag{14}$$

From these equations we can notice that

$$\begin{aligned} A_2 &= 2\int_{d-r_2}^{d_1} \sqrt{r_2^2 - (x-d)^2}dx \\ &= 2\int_{-r_2}^{d_1-d} \sqrt{r_2^2 - x^2}dx \\ &= 2\int_{d-d_1}^{r^2} \sqrt{r_2^2 - x^2}dx \\ &= 2\int_{d_2}^{r_2} \sqrt{r_2^2 - x^2}dx \end{aligned} \tag{15}$$

by using the fact that $d_2 = d - d_1$. The result is exactly the same equation than equation (12) if we change $d_1 \to d_2$ and $r_1 \to r_2$. Therefore if we compute A_1 first, we can obtain the result for A_2 as well.

$$\begin{aligned} A_1 &= 2\int_{d_1}^{r_1} \sqrt{r_1^2 - x^2}dx \\ &= 2r_1\int_{d_1}^{r_1} \sqrt{1 - \Big(\frac{x}{r_1}\Big)^2}dx \\ &= 2r_1\int_{d_1/r_1}^{1} \sqrt{1 - x^2}dx \end{aligned} \tag{16}$$

Now we have to integrate $\sqrt{1-x^2}$, which can be done following way:

$$\begin{aligned} \int \sqrt{1-x^2}dx &= x\sqrt{1-x^2} - \int x\Big(\frac{-x}{\sqrt{1-x^2}}\Big)dx \\ &= x\sqrt{1-x^2} + \int \frac{x^2-1}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx \\ &= x\sqrt{1-x^2} - \int \sqrt{1-x^2}dx + sin^{-1}(x) \end{aligned} \tag{17}$$

Which leads to:

$$\int \sqrt{1-x^2}dx = \frac{1}{2}\Big(x\sqrt{1-x^2} + sin^{-1}(x)\Big). \tag{18}$$

Now we can use the equation (18) on equation (16) which gives us:

$$\begin{aligned} A_1 &= r_1^2\Big(\frac{\pi}{2} - \frac{d_1}{r_1}\sqrt{1-\Big(\frac{d_1}{r_1}\Big)^2} - sin^{-1}\Big(\frac{d_1}{r_1}\Big)\Big) \\ &= r_1^2\Big(cos^{-1}\Big(\frac{d_1}{r_1}\Big) - \frac{d_1}{r_1}\sqrt{1-\Big(\frac{d_1}{r_1}\Big)^2}\Big) \\ &= r_1^2 cos^{-1}\Big(\frac{d_1}{r_1}\Big) - d_1\sqrt{r_1^2 - d_1^2} \end{aligned} \tag{19}$$

where the fact that $\pi/2 - sin^{-1}(\alpha) = cos^{-1}(\alpha)$ for any α from -1 to 1 is used. Now we get the result for A_2 by only doing substitutions $d_1 \to d_2$ and $r_1 \to r_2$ which gives us:

$$A_2 = r_2^2 cos^{-1}\Big(\frac{d_2}{r_2}\Big) - d_2\sqrt{r_2^2 - d_2^2}. \tag{20}$$

Therefore the intersection area of two circles is the sum of A_1 and A_2

$$A = r_1^2 cos^{-1}\Big(\frac{d_1}{r_1}\Big) - d_1\sqrt{r_1^2 - d_1^2} + r_2^2 cos^{-1}\Big(\frac{d_2}{r_2}\Big) - d_2\sqrt{r_2^2 - d_2^2} \tag{21}$$

where

$$\begin{aligned} d_1 &= \frac{r_1^2 - r_2^2 + d^2}{2d} \\ d_2 &= \frac{r_2^2 - r_1^2 + d^2}{2d} \end{aligned} \tag{22}$$

7.2 Appendix B

As explained earlier in thesis, we assume that $\theta = 0$ for ellipse. We also assume that planet does transit its host star the way that rings (ellipse) and the star (circle) has two intersection points. In a very rare cases the rings and the star could have 3 or 4 intersection points but because the size of the star is usually a lot bigger compared to the planet system's size we can neglect the possibility for more intersection points.

Therefore intersection area of circle and ellipse can be obtained quite straightforwardly; The equation of circle is known as

$$(x-a)^2 + (y-b)^2 = r^2 \tag{23}$$

where a and b are the center coordinates of a circle and r is circle's radius. We also know the equation of an ellipse as

$$\Big(\frac{x-x_0}{\alpha}\Big)^2 + \Big(\frac{y-y_0}{\beta}\Big)^2 = 1 \tag{24}$$

where x_0 and y_0 are the center coordinates of an ellipse, α is semi-major axis and β is the semi-minor axis.

Practically we know the center coordinates of a circle and an ellipse and also we know the semi-major and -minor axes. The only unknown variables are x and y which can be solved by doing system of equations. The result will be the intersection point coordinates which will be used to calculate the intersection area. As seen from the figure 30 the intersection area is divided to two areas. A_1

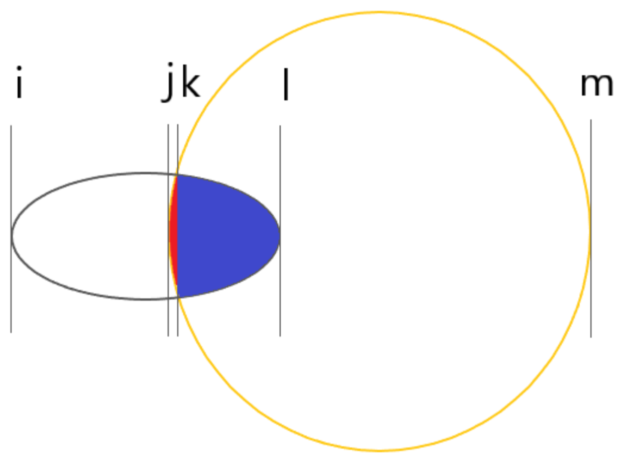


Figure 30: Intersection area of an ellipse and a circle and critical x-coordinates to compute it

is marked with red and A_2 with blue. A_1 is given by

$$\begin{aligned} A_1 = \Big(&\int_j^k y + \sqrt{-(-m+x)(-j+x)} dx \\ &- \int_j^k y - \sqrt{-(-m+x)(-j+x)} dx \Big) \end{aligned} \tag{25}$$

where j, k and m are x coordinates for the ultimate left point of the circle, intersection points and ultimate right point of the circle, respectively. A_2 is given by

$$A_2 = \Big(\int_k^l y + \frac{semiminor}{semimajor}\sqrt{-(-l+x)(-i+x)}dx \\ - \int_k^l y - \frac{semiminor}{semimajor}\sqrt{-(-l+x)(-i+x)}dx \Big) \tag{26}$$

where i and l are x coordinates for ultimate left point of the ellipse and ultimate right point of the ellipse, respectively. By summing up A_1 and A_2 we get the whole intersection area of circle and ellipse. As a note, we can calculate the integrals only if we know the x-coordinates for lines shown in figure 30. When producing the synthetic light curve with program made for this thesis, all those x coordinates are known all the time on every step.

7.3 Programs

In this section there is shown the programs made with python which can read light curve data from Kepler and K2 and produce synthetic light curves. The program made for K2 candidates is in figure 31 and the program for Kepler candidates in figure 32. The rest figures 33-40 are from the program made for the synthetic light curve production.

7.4 K2 Light Curve Program

```python
import csv
import math
import matplotlib.pyplot as plt

name = input("Name of the .csv file you want to plot? ")
#When running the program we can determine the file we want to examine
str = name
str2 = str.replace(".csv", "")
str3 = str2.replace("k2","K2-")
file = open(name, newline='')
reader = csv.reader(file)

header = next(reader)
#Skip the first row of .csv file (there is no data)

datax = []
datay = []

for row in reader:
    time = float(row[0])
    flux = float(row[1])

    if time >= 0:
        datax.append(time)
        datay.append(flux)
#Reads .csv files and classifies that on every row the first parameter
#is a time and the second parameter is a flux. Then adds the values for
#lists that they can be easily plotted

plt.plot(datax, datay, 'k', ms=2, label = "Transit light curve")
plt.title("Transit light curve of {}".format(str3))
plt.ylabel("Flux (corrected)")
plt.xlabel("Time (days)")
plt.show()
#Plots flux of a target in respect of time
```

Figure 31: Program which plots light curves from K2 data

7.5 Kepler Light Curve Program

```python
from astropy.io import fits
import matplotlib.pyplot as plt
import numpy as np

dvt_file = input("Name of the .fits file you want to plot? ")
#When running the program we can determine the file we want to examine
str = dvt_file
str2 = str.replace(".fits", "")
str3 = str2.replace("kepler","Kepler-")

with fits.open(dvt_file, mode="readonly") as hdulist:
    times = hdulist[1].data['TIME']
    phases = hdulist[1].data['PHASE']
    fluxes_init = hdulist[1].data['LC_INIT']
    model_fluxes_init = hdulist[1].data['MODEL_INIT']
 #Extracting columns needed for light curve.  Those are stored in the binary FITS table
 #in the first extension.  Extracting the timestamps in BKJD (Kepler Barycentric Julian Day),
 #phase, initial fluxes, and corresponding
 #model fluxes.

sort_indexes = np.argsort(phases)

fig, ax = plt.subplots(figsize=(10,5))
#Determining figure and axis size.

ax.plot(phases[sort_indexes], fluxes_init[sort_indexes], 'ko',
       markersize=0.5)
#Plotting the fluxes as black circles in sorted order
ax.plot(phases[sort_indexes], model_fluxes_init[sort_indexes], '-r')
#Modelling the flux as red line.

fig.suptitle('Folded Light Curve And Transit Model of {}'.format(str3))
ax.set_ylabel("Flux (relative)")
ax.set_xlabel("Orbital Phase")
#Plots a flux in respect of orbital phase
plt.show()
```

Figure 32: Program which plots light curves from Kepler data

7.6 Synthetic Light Curve Program

```
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
import math
from PIL import Image
from math import pi, cos, sin
import numpy as np
from matplotlib.patches import Ellipse
from matplotlib.widgets import Button
from itertools import product
import sympy as sp
from scipy.integrate import quad



#When running the program, you can choose some parameters and the rest are defined here

r_star = input("Hey! This program simulates a lightcurve from transit of a ringed planet. Start with entering the radius of a host star in kilometers: ")
r_planet = input("Now enter the radius of planet in kilometers: ")
nameoftheplanet = input("Enter the name of the planet: ")
angle = input("Give angle: ")
mean = [5000000, 5000000]  #star's center coordinates
meanx = 5000000-(1.5*int(r_star))   #planet systems starting x- coordinates
meany = 5000000  #planet systems starting y- coordinates
width = input("Now enter the diameter of outer rings in kilometers: ")
semimajor = int(width) / 2
inclination = input("Now enter the inclination angle: ")
height = math.sin(math.radians(90-int(inclination)))*int(width)
semiminor = int(height) / 2
ratio = int(height) / int(width)  # Needed for ellipse's integration
distance_from_surface = input("Now enter the distance of inner rings from the surface of planet in kilometers in x-axis: ")
width2 = 2*int(r_planet) + 2*int(distance_from_surface)
semimajor2 = width2 / 2
height2 =  ratio * width2
semiminor2 = height2 / 2
Tau = 0.1  #optical depth of a rings
horizontalrings = semimajor
horizontalrings2 = semimajor2
orbital_period = input("Enter the orbital period in days: ")
orbital_period2 = float(orbital_period) * 24 * 60 * 60  # Orbital period in seconds
semimajoraxisofplanet = input("Enter the distance between a planet and its host star in AU's: ")
orbitlength = 2 * math.pi * float(semimajoraxisofplanet) * 149597871
orbital_speed = orbitlength / orbital_period2 #km/s
orbital_phase = orbitlength / 360 #one degree length

# Here are definitions for star, planet and rings and definitions for visuals
```

Figure 33: Program which creates synthetic light curve

```
# Here are definitions for star, planet and rings and definitions for visuals

star = plt.Circle((mean), int(r_star), facecolor = "white")
planet = plt.Circle((meanx,meany), int(r_planet), color = "black")
external_ring = mpl.patches.Ellipse(xy=[meanx, meany], width=int(width), height=int(height), angle = 180+int(angle), edgecolor='gray', lw=0.1, facecolor=
inner_ring = mpl.patches.Ellipse(xy=[meanx, meany], width=width2, height=height2, angle = 180+int(angle), edgecolor='gray', lw=0.1, facecolor='white')

s_area = pi * int(r_star)**2  #Area of a star

#Plotting parameters
fig, ax = plt.subplots()
ax.set(xlim = (0, 10000000), ylim = (0, 10000000))
ax.set_ylabel("Million kilometers")
ax.set_xlabel("Million kilometers")

background = plt.fill_between((0, 10000000), 0, 10000000 ,color='black')
ax.add_artist(star)
ax.add_artist(external_ring)
ax.add_artist(inner_ring)
ax.add_artist(planet)
ax.set_aspect('equal')

# Definitions for Move and Pic buttons
button2 = Button(plt.axes([0.1, 0.025, 0.1, 0.04]), 'PIC', color='white', hovercolor='0.975')
button = Button(plt.axes([0.8, 0.025, 0.1, 0.04]), 'Move', color='white', hovercolor='0.975')

# Definitions for areas of planet, inner ring and outer ring
A_planet = math.pi*(int(r_planet)**2)
A_ellipse = (math.pi * semimajor * semiminor)
A_ellipse2 = (math.pi * semimajor2 * semiminor2)

factor = 1 - math.exp(-float(Tau) / math.cos(math.radians(int(inclination)))) # beta-factor of rings
#empty lists for integration results etc.
factorlist = []
meanx2 = 5000000-(1.5*int(r_star))
steps = []
planetarea = []
ringarea = []
ringarea2 = []
steps2 = 0
norm = []
star_area = []
ones = []
phasesteps = []
phase = 0
```

Figure 34: Program which creates synthetic light curve

```python
#This function moves the planet system every time "Move" is clicked
#It also adds some values for lists which are needed later in integration
#This function finally calculates the intersection area of a planet and star

def intersectionPS(event):
    global meanx2
    global steps2
    global phase
    meanx2 += 0.1*int(r_planet)
    steps2 += ((0.1*int(r_planet))/orbital_speed)/(3600*24)
    steps.append(steps2)
    phase += 0.1*int(r_planet)/orbital_phase
    phasesteps.append(phase)
    star_area.append(s_area)
    ones.append(1)
    factorlist.append(factor)
    distance = abs(5000000 - meanx2)
    d1 = (int(r_star)**2 - int(r_planet)**2 + distance**2) / (2*distance)
    d2 = distance - d1
    if distance >= int(r_star) + int(r_planet):
        planet_star_intersectionarea = 0
        planetarea.append(planet_star_intersectionarea)
    elif distance < int(r_star) + int(r_planet) and distance >= int(r_star) - int(r_planet):
        planet_star_intersectionarea = int(r_star)**2*math.acos(d1/int(r_star)) - d1*(int(r_star)**2-d1**2)**(1/2) + int(r_planet)**2*math.acos(d2/int(r_
        planetarea.append(planet_star_intersectionarea)
    else:
        planet_star_intersectionarea = A_planet
        planetarea.append(planet_star_intersectionarea)

#This function re-draws the planet system everytime it's moving in a plot
def move(event):
    global meanx2
    planet2 = plt.Circle((meanx2,meany), int(r_planet), color = "black")
    external_ring2 = mpl.patches.Ellipse(xy=[meanx2, meany], width=int(width), height=int(height), angle = 180+int(angle), edgecolor='gray', lw=0.1, face
    inner_ring2 = mpl.patches.Ellipse(xy=[meanx2, meany], width=width2, height=height2, angle = 180+int(angle), edgecolor='gray', lw=0.1, facecolor='whi
    ax.add_artist(background), ax.add_artist(star), ax.add_artist(external_ring2), ax.add_artist(inner_ring2), ax.add_artist(planet2)



#These are the integrals for rings-star intersection area
##########################################################
def integration1(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
```

Figure 35: Program which creates synthetic light curve

```python
def integration1(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
    return 5000000 + ((-(-mostrightpointofstar+x)*(-mostleftpointofstar+x))**(1/2))

def integration2(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
    return 5000000 - ((-(-mostrightpointofstar+x)*(-mostleftpointofstar+x))**(1/2))

def integration3(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
    return 5000000 + (ratio)*((-(-mostrightpointofrings+x)*(-mostleftpointofrings+x))**(1/2))

def integration4(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
    return 5000000 - (ratio)*((-(-mostrightpointofrings+x)*(-mostleftpointofrings+x))**(1/2))

def integration5(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
    return 5000000 + ((-(-mostrightpointofstar+x)*(-mostleftpointofstar+x))**(1/2))

def integration6(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
    return 5000000 - ((-(-mostrightpointofstar+x)*(-mostleftpointofstar+x))**(1/2))

def integration7(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
```

Figure 36: Program which creates synthetic light curve

```python
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
    return 5000000 + (ratio)*((-(-mostrightpointofrings2+x)*(-mostleftpointofrings2+x))**(1/2))

def integration8(x):
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
    return 5000000 - (ratio)*((-(-mostrightpointofrings2+x)*(-mostleftpointofrings2+x))**(1/2))
##########################################################

#These two next functions define the intersection area of a planet and rings
def integrationofpr(x):
    return (((int(r_planet)**2-x**2)**(1/2)) - ((semiminor/semimajor)*((semimajor**2-x**2)**(1/2))))

def integrationpr(event):
    x, y = sp.symbols('x, y',real=True)
    eq1 = sp.Eq(x**2 + y**2, int(r_planet)**2)
    eq2 = sp.Eq(((x)/semimajor)**2 + ((y)/semiminor)**2, 1)
    answer2 = sp.solve((eq1, eq2), (x, y))
    answer = [tuple([i.n(14) for i in j]) for j in answer2]
    xa = answer[3][0]
    ya = answer[3][1]
    minimum = 0
    maximum = xa
    anspr, errpr = quad(integrationofpr, minimum, maximum)
    areaofplanetrings = (math.pi * int(r_planet)**2) - 4*anspr

#This function calculates the intersection area of outer and star every time the system is moving
def intersectionpoints(event):
    distance = abs(5000000 - meanx2)
    d1 = (int(r_star)**2 - int(r_planet)**2 + distance**2) / (2*distance)
    d2 = distance - d1
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings = int(meanx2 + horizontalrings)
    mostleftpointofrings = int(meanx2 - horizontalrings)
    if int(r_star) + horizontalrings <= distance:
        ringarea.append(0)
    elif int(r_star) > distance + horizontalrings:
        ringarea.append(A_ellipse)
    elif int(r_star) < distance + horizontalrings and distance - horizontalrings < int(r_star) and meanx2 < 5000000:
        x, y = sp.symbols('x, y',real=True)
        eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
```

Figure 37: Program which creates synthetic light curve

```python
mostrightpointofrings = int(meanx2 + horizontalrings)
mostleftpointofrings = int(meanx2 - horizontalrings)
if int(r_star) + horizontalrings <= distance:
    ringarea.append(0)
elif int(r_star) > distance + horizontalrings:
    ringarea.append(A_ellipse)
elif int(r_star) < distance + horizontalrings and distance - horizontalrings < int(r_star) and meanx2 < 5000000:
    x, y = sp.symbols('x, y',real=True)
    eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
    eq2 = sp.Eq(((x-meanx2)/semimajor)**2 + ((y - 5000000)/semiminor)**2, 1)
    answer2 = sp.solve((eq1, eq2), (x, y))
    answer = [tuple([i.n(14) for i in j]) for j in answer2]
    int_x1 = answer[0][0]
    int_y1 = answer[0][1]
    int_x2 = answer[1][0]
    int_y2 = answer[1][1]
    minimum1 = mostleftpointofstar
    maximum1 = int_x1
    minimum2 = int_x2
    maximum2 = mostrightpointofrings
    ans1, err1 = quad(integration1, minimum1, maximum1)
    ans2, err2 = quad(integration2, minimum1, maximum1)
    ans3, err3 = quad(integration3, minimum2, maximum2)
    ans4, err4 = quad(integration4, minimum2, maximum2)
    isection_rings_star = ((ans1-ans2)+(ans3-ans4))
    ringarea.append(isection_rings_star)
else:
    x, y = sp.symbols('x, y',real=True)
    eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
    eq2 = sp.Eq(((x-meanx2)/semimajor)**2 + ((y - 5000000)/semiminor)**2, 1)
    answer2 = sp.solve((eq1, eq2), (x, y))
    answer = [tuple([i.n(14) for i in j]) for j in answer2]
    int_x1 = answer[0][0]
    int_y1 = answer[0][1]
    int_x2 = answer[1][0]
    int_y2 = answer[1][1]
    minimum1 = mostrightpointofstar
    maximum1 = int_x1
    minimum2 = int_x2
    maximum2 = mostleftpointofrings
    ans1, err1 = quad(integration1, minimum1, maximum1)
    ans2, err2 = quad(integration2, minimum1, maximum1)
    ans3, err3 = quad(integration3, minimum2, maximum2)
    ans4, err4 = quad(integration4, minimum2, maximum2)
    isection_rings_star = abs((ans1-ans2)+(ans3-ans4))
    ringarea.append(isection_rings_star)
```

Figure 38: Program which creates synthetic light curve

```python
#This function calculates the intersection area of inner ring and star every time the system is moving
def intersectionpoints2(event):
    distance = abs(5000000 - meanx2)
    mostrightpointofstar =  5000000 + int(r_star)
    mostleftpointofstar = 5000000 - int(r_star)
    mostrightpointofrings2 = int(meanx2 + horizontalrings2)
    mostleftpointofrings2 = int(meanx2 - horizontalrings2)
    d1 = (int(r_star)**2 - int(r_planet)**2 + distance**2) / (2*distance)
    d2 = distance - d1
    if int(r_star) + horizontalrings2 <= distance:
        ringarea2.append(0)
    elif int(r_star) > distance + horizontalrings2:
        ringarea2.append(A_ellipse2)
    elif int(r_star) < distance + horizontalrings2 and distance - horizontalrings2 < int(r_star) and distance > int(r_star) + int(r_planet) and distance
        x, y = sp.symbols('x, y',real=True)
        eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
        eq2 = sp.Eq(((x-meanx2)/semimajor2)**2 + ((y - 5000000)/semiminor2)**2, 1)
        answer2 = sp.solve((eq1, eq2), (x, y))
        answer = [tuple([i.n(14) for i in j]) for j in answer2]
        int_x1 = answer[0][0]
        int_y1 = answer[0][1]
        int_x2 = answer[1][0]
        int_y2 = answer[1][1]
        minimum1 = mostleftpointofstar
        maximum1 = int_x1
        minimum2 = int_x2
        maximum2 = mostrightpointofrings2
        ans1, err1 = quad(integration5, minimum1, maximum1)
        ans2, err2 = quad(integration6, minimum1, maximum1)
        ans3, err3 = quad(integration7, minimum2, maximum2)
        ans4, err4 = quad(integration8, minimum2, maximum2)
        isection_rings_star2 = ((ans1-ans2)+(ans3-ans4))
        ringarea2.append(isection_rings_star2)
    else:
        x, y = sp.symbols('x, y',real=True)
        eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
        eq2 = sp.Eq(((x-meanx2)/semimajor2)**2 + ((y - 5000000)/semiminor2)**2, 1)
        answer2 = sp.solve((eq1, eq2), (x, y))
        answer = [tuple([i.n(14) for i in j]) for j in answer2]
        int_x1 = answer[0][0]
        int_y1 = answer[0][1]
        int_x2 = answer[1][0]
        int_y2 = answer[1][1]
        minimum1 = mostrightpointofstar
        maximum1 = int_x1
        minimum2 = int_x2
```

Figure 39: Program which creates synthetic light curve

```python
            ans4, err4 = quad(integration8, minimum2, maximum2)
            isection_rings_star2 = ((ans1-ans2)+(ans3-ans4))
            ringarea2.append(isection_rings_star2)
        else:
            x, y = sp.symbols('x, y',real=True)
            eq1 = sp.Eq((x - 5000000)**2 + (y - 5000000)**2, int(r_star)**2)
            eq2 = sp.Eq(((x-meanx2)/semimajor2)**2 + ((y - 5000000)/semiminor2)**2, 1)
            answer2 = sp.solve((eq1, eq2), (x, y))
            answer = [tuple([i.n(14) for i in j]) for j in answer2]
            int_x1 = answer[0][0]
            int_y1 = answer[0][1]
            int_x2 = answer[1][0]
            int_y2 = answer[1][1]
            minimum1 = mostrightpointofstar
            maximum1 = int_x1
            minimum2 = int_x2
            maximum2 = mostleftpointofrings2
            ans1, err1 = quad(integration5, minimum1, maximum1)
            ans2, err2 = quad(integration6, minimum1, maximum1)
            ans3, err3 = quad(integration7, minimum2, maximum2)
            ans4, err4 = quad(integration8, minimum2, maximum2)
            isection_rings_star2 = abs((ans1-ans2)+(ans3-ans4))
            ringarea2.append(isection_rings_star2)

#This function draws the synthetic light curve when pressing "PIC"
def Lightcurve(event):
    ringarea_real2 = [x1 - x2 for (x1, x2) in zip(ringarea, planetarea)]
    ringarea_real = [x1 * x2 for (x1, x2) in zip(ringarea_real2, factorlist)]
    blockingarea = [x1 + x2 for (x1, x2) in zip(ringarea_real, planetarea)]
    normalized_1 = [x1/x2 for (x1, x2) in zip(blockingarea, star_area)]
    normalized = [x1 - x2 for (x1, x2) in zip(ones, normalized_1)]
    plt.figure()
    plt.plot(steps, normalized)
    plt.title("Synthetic transit light curve of {}".format(nameoftheplanet))
    plt.ylabel("Normalized flux (F/F_STAR)")
    plt.xlabel("Orbital phase (days)")
    #plt.xlabel("Time (days)")
    plt.show()

button.on_clicked(intersectionPS)
button.on_clicked(move)
button2.on_clicked(Lightcurve)
button.on_clicked(intersectionpoints)
button.on_clicked(intersectionpoints2)

plt.show()
```

Figure 40: Program which creates synthetic light curve

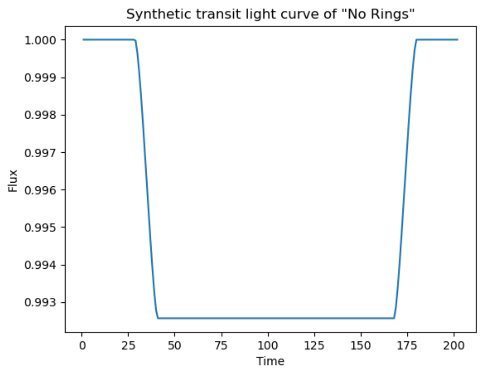
7.7 Test of the Model

The functionality of the Transit light curve model were tested with multiple sanity tests.

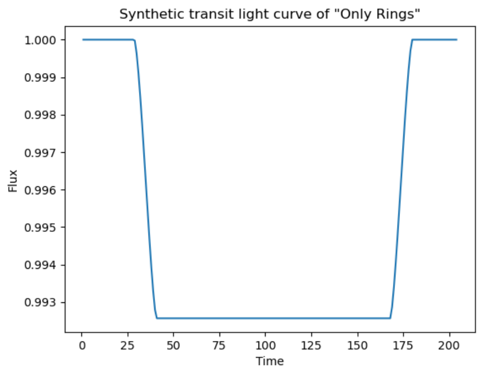
The first sanity test compares transits of the planet without rings and the rings without planet. We set rings' radius to be equal to planet's radius, inclination angle i_R to be 0 (rings are face-on) and optical depth $\tau = 10$. With that optical depth the transparency of the rings should be zero and therefore the effect of the rings in transit the same than for just a planet. As we can see from the figure 41 both light curves are identical as they should be and therefore the first sanity test is successful.

Next sanity test compares different optical depths τ. When τ increases, should the transit depth become deeper since rings do block more light from the star. As seen in figure 42 the transit depth behaves just as it should and therefore the second sanity test is successful.

Finally we want to check if surface integrals are working properly. Let's have a situation where planet system has only rings (inner edge set to 0) and is almost entered to the main phase of transit. In that case the intersection area of the whole planet system should be nearly its own surface area. Then we do the same for only planet without rings. For the first test we set $R_{planet} = 0km$, $R_{rings} = 140000km$



(a)



(b)

Figure 41: Transit light curve of planet without rings ($R_{planet} = 60000, R_{star} = 696000, R_{rings} = 0, \tau = 10, \theta = 0, i_R = 0$) in (a).
Transit light curve of rings without planet ($R_{planet} = 0, R_{star} = 696000, R_{rings} = 60000, \tau = 10, \theta = 0, i_R = 0$) in (b)

(semi-major axis 140 000 km and semi-minor axis 75 000km), $R_{star} = 696000km$ and for the second test we set $R_{planet} = 60000km, R_{rings} = 0km, R_{star} = 696000km$.

The surface area of the circle with radius of 60000 km can be obtained as

$$A_{planet} = \pi * R_{planet}^2 = \pi * (60000km)^2 = 11309733552km^2. \tag{27}$$

We can clearly see from the figure 43 that sanity test is successful for a planet, because given intersection area from program is 11 089 219 320 km^2 and almost the area of the planet itself.

The surface area of the ellipse with semi-major axis of 140 000 km and semi-minor axis of 75 000 km can be obtained from

$$A_{rings} = \pi * a * b = \pi * 140000km * 75000km = 32986722862km^2. \tag{28}$$

We can notice from figure 44 that sanity test is also successful for rings, since given intersection area from program is 32 715 728 271 km^2 and almost the area of the just rings.

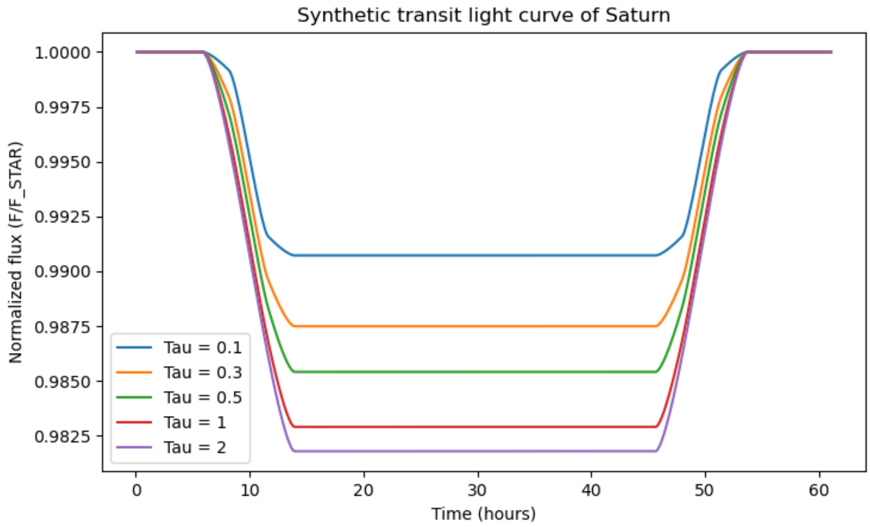


Figure 42: Transit light curve of planet with various values for τ. $i_R = 63°$

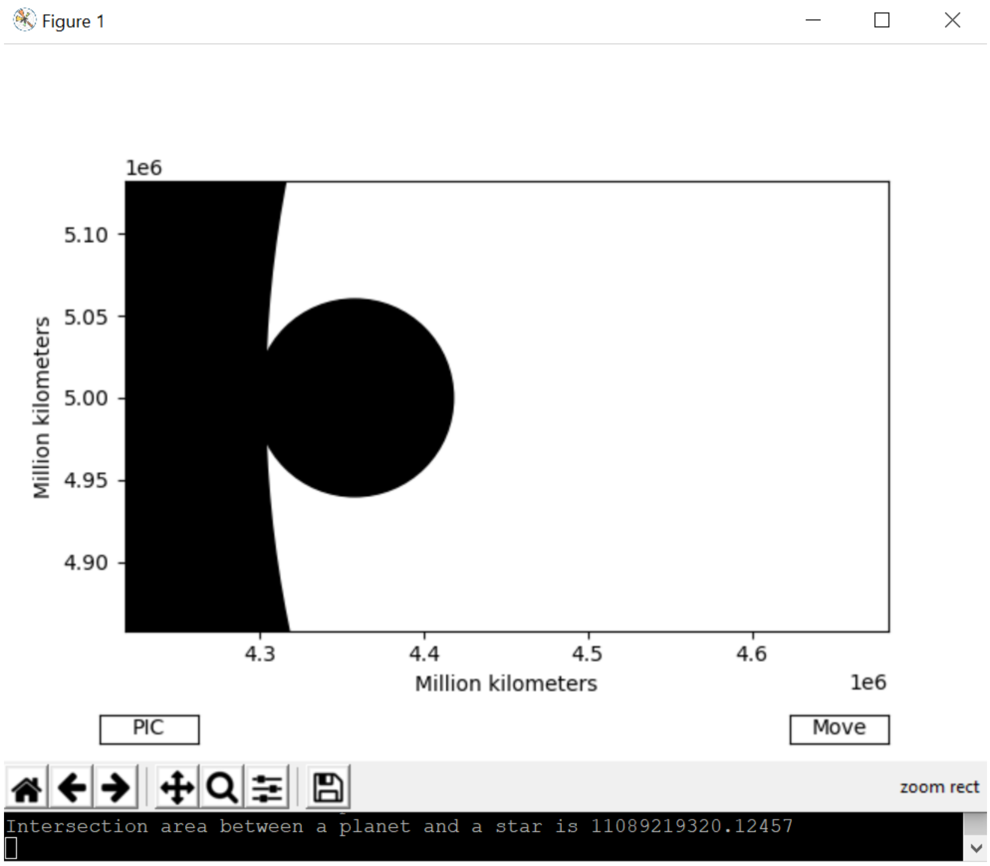


Figure 43: Intersection area of planet-star

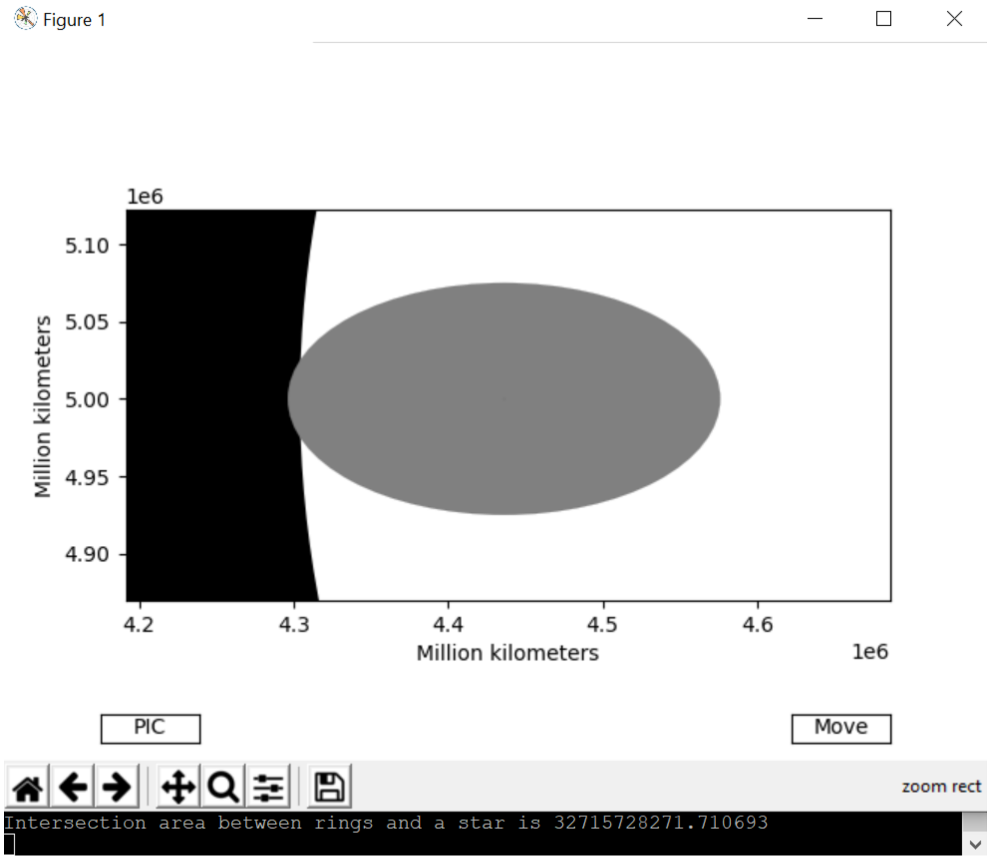


Figure 44: Intersection area of rings-star

8 References

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